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A TWO-STAGE MODELING FRAMEWORK FOR THE OPTIMAL UTILIZATION OF LOAD FLEXIBILITY IN ENERGY AND RESERVE MARKETS

TESIS PARA OPTAR AL GRADO DE MAGÍSTER EN CIENCIAS DE LA INGENIERÍA, MENCIÓN ELÉCTRICA

MEMORIA PARA OPTAR AL TÍTULO DE INGENIERO CIVIL ELÉCTRICO

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RESUMEN DE LA MEMORIA PARA OPTAR AL TÍTULO DE INGENIERO CIVIL ELÉCTRICO Y DE LA TESIS PARA OPTAR AL GRADO DE MAGÍSTER EN CIENCIAS DE LA INGENIERÍA, MENCIÓN ELÉCTRICA
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FECHA: 2021
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ESQUEMA DE MODELAMIENTO DE DOS ETAPAS PARA LA UTILIZACIÓN ÓPTIMA DE CARGAS FLEXIBLES EN MERCADOS DE ENERGÍA Y RESERVA

Facilitar nuevos negocios para que los comercializadores puedan ofrecer múltiples servicios de respuesta de demanda (RD) es clave para lograr una profunda descarbonización. Estos nuevos negocios tienen la capacidad de mejorar la flexibilidad del sistema, como también generar ingresos adicionales. Sin embargo, resulta fundamental una gestión eficiente de las cargas flexibles para poder ofrecer múltiples servicios al sistema. Por lo tanto, este trabajo propone un *framework* de dos etapas que busca optimizar la operación de un comercializador que ofrece servicios de RD en los mercados de energía y reserva. En la primera etapa, se utiliza Optimización Inversa (OI) para caracterizar el comportamiento del portafolio de cargas flexibles. La segunda etapa busca optimizar la operación del comercializador, el cual es formulado mediante un problema de optimización binivel entre el comercializador y el mercado de reserva, en donde el comercializador además se encuentra restringido por los problemas de optimización de las cargas. Utilizando el mercado Chileno como referencia, este trabajo demuestra los beneficios económicos de un enfoque multi-servicio comparándolo con el enfoque actual de sólo participar en el mercado de energía. Además, se demuestran los beneficios de permitir RD en los mercados de reserva a partir de la reducción del costo asociado.

RESUMEN DE LA MEMORIA PARA OPTAR AL TÍTULO DE INGENIERO CIVIL ELÉCTRICO Y DE LA TESIS PARA OPTAR AL GRADO DE MAGÍSTER EN CIENCIAS DE LA INGENIERÍA, MENCIÓN ELÉCTRICA
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A TWO-STAGE MODELING FRAMEWORK FOR THE OPTIMAL UTILIZATION OF LOAD FLEXIBILITY IN ENERGY AND RESERVE MARKETS

Facilitating new businesses for retailers to unlock Demand Response (DR) services in electricity markets is key to enabling deep decarbonization. These new businesses will create additional revenue streams for retailers as well as enhance system flexibility. However, it will require retailers to appropriately manage their contracted flexible loads to be able to offer multiple services to the system. Therefore, this work proposes a two-stage modeling framework to determine the optimal operation of a retailer that provides DR services in energy and reserve markets. In the first stage, a data-driven estimation method based on Inverse Optimization (IO) is used to characterize consumer behavior for future operating scenarios. The results are inputted into the second stage, where the model optimizes retailers' operations by formulating a bilevel optimization problem between the retailer and the reserve market in which the retailer is also constrained by loads' optimization problems. Through several case studies based on the Chilean electricity market, we demonstrate the economic benefits of a multi-service approach by comparing it against the current approach of an actual retailer that only participates in the energy market. Additionally, we also demonstrate the benefits of enabling DR in reserve markets by quantifying reserve cost savings.

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Chapter 1

Introduction

1.1 Motivation

The envisaged high share of Renewable Energy Sources (RES) of future power systems will impose significant challenges when maintaining system reliability and energy security. In fact, as more RES are integrated into the system, increased uncertainty and variability derived from these sources will induce significant stress when maintaining system balance through generation-side flexibility alone. Furthermore, the impending shift from inertia-based generation will force the need to find alternative sources that are capable of providing the same level of operating reserves as current fossil-fueled generators [1].

In this context, where more operational flexibility is required, Demand Response (DR) emerges as an attractive alternative. This is due to the ability to redistribute or curtail consumers' demand without the need for capital-heavy investments. DR recognized benefits include peak load reduction during severe grid conditions [2], congestion management [3], and the ability to defer investments in distribution and transmission infrastructure [4]. Moreover, DR can be used to increase the efficiency of electricity markets by reducing peak demand and therefore minimizing the use of marginal cost generating units [5].

Within the realm of reserve provision, recent advances in control and communication technologies (CCT) have increased the accuracy and speed of DR actions, making it suitable to fully participate in the reserve market. Nowadays, DR can provide cheap and cost-efficient operating reserves, without the need to use partially loaded thermal generating units [6, 7, 8]. This additional flexibility introduced by DR allows Independent System Operators (ISOs) to have a greater portfolio of security services when maintaining system reliability, thus improving the efficiency of contingency management and power system operation [9].

However, market constraints and regulatory barriers may prevent these agents from diversifying their array of DR-based services despite having the necessary infrastructure to participate in different markets [10]. Furthermore, while some countries are modifying their market designs to enable demand-side participation in the reserve market [1, 11, 12, 13], there is still high uncertainty for retailers as to whether offering multiple DR-based services is sufficiently profitable compared to solely using their DR capabilities to reduce procurement costs in the energy market. It is, therefore, paramount to analyze the economic benefits that retailers can achieve when they, in addition to utilizing their flexible load portfolio in the energy market, opt to participate in the reserve market and how this participation could impact market conditions and system-wide reserve costs.

1.2 Hypothesis

The hypothesis stated in this work is that electricity retailers can attain significant economic benefits when offering DR-based services in energy and reserve markets, while also reducing total reserve costs to the system and allowing ISOs to fully exploit the reliability benefits of unlocking DR-based solutions in power system operation.

1.3 General Objective

Design, implement and solve a modeling framework that optimizes the operation of an electricity retailer that provides DR-based services in day-ahead energy and reserve wholesale markets. This modeling framework should be able to, first, capture the market equilibrium between the electricity retailer and the ISO, and second, accurately characterize the behavior of flexible loads within the portfolio of the electricity retailer.

1.4 Specific Objectives

- Analyze the economic performance perceived by electricity retailers when utilizing its load flexibility to offer DR-based services in both energy and reserve markets, and compare the results against a benchmark model that solely participates in the energy market.
- Analyze the impacts upon enabling demand-side participation in the reserve markets, and how it can affect market prices and overall reserve costs.
- Carry out sensitivity analyses on reserve market activation rates, and flexible load contract volumes to validate the robustness of the proposed two-stage framework.
- Demonstrate the advantages of using a data-driven load characterization algorithm based on inverse optimization by comparing the results with other commonly used methods.
- Demonstrate the realistic applicability of the proposed modeling framework, through the use of real world data corresponding to the Chilean electricity market.

1.5 Contributions

- Develop a two-stage, data-driven modeling framework for retailers that first, appropriately captures load behavior and future decision-making patterns, and second, optimally manages their flexible load portfolio through the offering of DR-based services in energy and reserve markets.

- Demonstrate that electricity retailers can attain significant economic benefits when offering DR-based services in energy and reserve markets, instead of solely participating in the energy market.
- Quantify the reserve cost reduction to the system upon enabling demand-side participation in the reserve market.
- Illustrate the benefits and fitting capabilities of using inverse optimization to characterize load behavior, and show that, by using a data-driven algorithm, our analysis is based on a realistic modeling approach, rather than the commonly used method of using arbitrary functions, predefined parameters or demand elasticities for load modeling.

1.6 Structure of the Document

This document is structured in four main chapters. Chapter 1 states the motivations and objectives of this work. A review of international experience regarding the utilization of DR-based services in energy and reserve markets, as well as a literature review on the fundamental concepts that will be used throughout this work is presented in Chapter 2. Chapter 3 presents the mathematical formulation of the two-stage modeling framework. Chapter 4 presents the case study based on the Chilean electricity market. Finally, Chapter 5 states the main conclusions and future work is proposed.

Chapter 2

State of the Art: Best Practices and Literature Review of Demand Response Services in Energy and Reserve Markets

2.1 Best Practices and International Experience

Enabling DR-based services in energy and reserve markets can greatly enhance the efficiency of power system operation, due to the ability to defer investments in transmission and distribution assets [4], peak shaving during high-priced hours [5], congestion management [3], and the overall improvement in the efficiency of electricity markets [9]. Moreover, recent advances in control and communication technologies (CCT) allow DR to be utilized as reserves in a cheap and cost-efficient manner, hence avoiding the use of partially loaded thermal generating units and providing an additional source of flexibility for contingency management [6, 7, 8]. However, one of the salient features of demand-side participation in electricity markets worldwide is the lack of standardization between regions and the high disparity in its adoption. While some countries have opened most of their electricity markets to DR-based services, others remain completely closed or present small participation rates due to high regulatory barriers (such as minimum market entry requirements of several MW or the prohibition of demand-side aggregation), lack of financial incentives and high uncertainty associated with the operational benefits of enabling DR [11, 12, 13]. It is therefore important to analyze the best practices and the international experience regarding DR-based services to first, assess current trends and examine the key elements that drive its adoption throughout different regions, second, establish a common baseline for analyzing the main contributions in the following literature review and third, determine the applicability and scalability of a multi-service approach for retailers.

In the United States and Canada, multiple system operators have opened their markets and allowed DR providers to fully offer its services alongside generation-based resources. Traditionally, DR was used as an energy resource for load reduction during extremely high-priced hours. The Pennsylvania-New Jersey-Maryland Interconnection (PJM), for example, has enabled DR participation in the wholesale energy markets since 2002, where DR was used as an emergency resource when wholesale energy prices raised above 75 [$\frac{\$USD}{MWh}$] [11]. Similarly, the New England ISO (NE-ISO) has carried out voluntary DR programs for energy market prices above 100 [$\frac{\$USD}{MWh}$] [14].

Based on the positive results when using DR to reduce energy prices and the associated energy cost savings, several ISOs in the United States started carrying out DR programs in their energy markets. Such examples include the New York ISO (NYISO) [15], which currently allows DR providers to fully participate in the day-ahead energy market and the California Independent System Operator (CAISO), which currently has over 1,612 [MW] of DR resources enlisted in load reduction programs [11]. Additionally, in 2011 the Federal Energy Regulatory Commission (FERC) under order 745 established a unified guideline with the goal to standardize DR participation in the energy markets and to encourage other ISOs to begin integrating DR as an energy resource [16].

Besides DR-based services in the energy markets, most ISOs in the region have opened their reserve markets for DR. For example, ISOs such as the Midcontinent Independent System Operator (MISO) [11], the Electric Reliability Council of Texas (ERCOT) [17] and NYISO [15] have enabled DR to participate in most contingency reserve services, such as non-spinning, supplementary and regulation reserves. PJM has one of the highest levels of DR-based services offered in reserve markets, where DR provides over 25% in Day-Ahead scheduling reserves and 31% in synchronized (spinning) reserves [18]. In Canada, most of the states are vertically regulated and Manitoba is under the jurisdiction of MISO. However, the ISO of Ontario (IESO) is currently developing a pilot project to assess the provision of ancillary services (including reserves) by DR [11].

Despite several DR initiatives with overall good results, one of the biggest challenges faced by the region regarding the full-scale integration of relies on its high-risk business model. In many energy systems of the region, customers that decide to utilize its flexibility to offer DR-based services in the energy or reserve markets are paid in an event-based fashion, resulting in the reliance on unpredictable and infrequent events for revenue [11]. Therefore, it is envisaged that DR adoption will be greatest in those systems that are capable of providing steady revenue streams to their flexible consumers, either through regular capacity payments, or by allowing DR to participate in multiple types of services (such as energy, reserves and other ancillary services).

In the case of Europe, this region has greatly advanced towards the integration and utilization of DR-based services in energy and reserve markets. However, there is still high disparity between countries within the region. In regards to reserve provision, many European countries have opened their markets for DR and made the necessary adjustments to enable demand-side participation. For example, countries such as Belgium, the Netherlands, Germany, Great Britain, Switzerland, France and the Nordic region have opened most of their reserve markets to enable DR-based services [12]. Within this group, France and Switzerland are the most advanced regarding DR integration, having developed detailed frameworks for demand-side participation, including the standardization of roles and responsibilities of market participants [12, 19]. However, there are still some countries in the region (such as Spain, Portugal, Poland, among others) that do not allow for flexible loads to provide DR in their reserve markets, while others that do allow it (such as Denmark) currently face significant regulatory barriers that prevent large-scale integration [12, 13].

In regards to DR-based services in the wholesale energy markets, the high disparity between European countries is still present. While some countries (such as the ones located in the Nordic region) enable large-sized and aggregated consumers to independently trade their flexibility in their wholesale markets, others (such as Spain, Portugal, Croatia, Bulgaria, amonth others) remain completely closed to DR [13], therefore losing an attractive opportunity for large-sized and aggregated consumers to lower their procurement costs. Additionally, other countries such as Great Britain, Finland or Germany allow DR in their wholesale markets, but only under the jurisdiction of their electricity retailer or balance responsible party (BRP) [12]. Specifically, these countries do not allow for independent DR aggregators to participate in the market and Great Britain is the only one listed above that remains open for a few large-sized consumers. Finally, there are some countries that technically allow for DR participation, but lack regulation or further regulatory incentives to enhance the adoption of DR. Such examples can be seen in Estonia, Italy, Slovenia and Poland [12].

Therefore, some of the biggest challenges faced by the region regarding the adoption and acceptace of DR-based services is the presence of high regulatory barriers and the partial restriction of several demand-side resources (load aggregators, large-sized consumers, households, etc.) in their electricity markets [12]. Despite several European countries opening their markets to DR, there are still high regulatory barriers in the region, where consumers in some countries face significant penalties and high product requirements (such as minimum bidding sizes, infrastructure requirements and response rates) when deciding to offer their flexibility as DR. Additionally, there is a lack of competition and the necessary market dynamics to fully exploit the potential of DR. Several countries prohibit demand-side aggregation or only allow retailers/BRPs to provide DR services in behalf of their consumers. Hence, policymakers and regulatory entities are stressing the need to further open the electricity markets in the region for DR aggregators and large-sized consumers in order to compete alongside retailers/BRPS and contribute to the overall efficiency of the market [12].

In regards to Latin America, this region is expected to increase the integration of DR and demand-side solutions in the near future. Currently, Mexico, Chile and Brazil are implementing large-scale smart metering roll-outs to improve their metering infrastructure and advance towards better demand-side management [1]. Brazil, Chile and Mexico are also pushing forward several DR pilot projects, such as the AES Electropaulo's smart grid program in Brazil [20], which aims to implement DR-based solutions in order to improve the efficiency of the distribution system. In addition to pilot projects and the improvement of metering infrastructure, countries such as Colombia and Chile also indirectly allow for DR participation in their electricity markets, through the establishment of bilateral contracts between generators and unregulated large-scale consumers that can include demand reduction during peak hours [21, 22].

However, in regards to market design and regulation, Latin America as a whole is still on its first steps towards a large-scale integration of DR. With a few relevant exceptions (such as Mexico, Chile and Brazil), most countries have not yet implemented any large-scale investment programs to upgrade their metering infrastructure, nor relevant pilot projects to test the potential benefits of DR. Moreover, to date, no country in the region has adjusted their electricity market design to fully enable direct demand-side participation [1]. Despite several policies targeted towards improving demand-side management, Latin American countries still lag behind other regions, presenting significant market and regulatory barriers such as prohibiting demand-side aggregation, high bidding/capacity requirements for market entry and lack of financial certainty regarding potential revenue streams. It is therefore important to study and analyze the contribution of enabling DR-based services in energy and reserve markets in the region, so that stakeholders and policymakers can assess the benefits and risks upon integrating demand-side resources such as DR in their power system operation.

2.2 Literature Review

The review of current international experience shows that several regions worldwide are pursuing initiatives to enhance the offering of multiple DR-based services in their energy and reserve markets. However, there are still several financial, behavioral and regulatory barriers that delay a system-wide adoption of demand-side solutions. In this vein, several works have studied how to efficiently integrate DR in energy and reserve markets and what are the potential benefits of offering multiple DR-based services to the system. One of the first publications that introduced the concept of DR in energy and reserve markets was [23], which proposed a joint energy and reserve dispatch model that included both generator and demand-side reserve offers and demonstrated that using DR as an energy and reserve resource generates significant economic benefits while also reducing the the market power of generators. Driven by the aforementioned benefits of DR, works such as [2, 24, 25] further analyzed the applications of DR in energy and reserve markets beyond economic dispatch. In particular, [2] proposed a stochastic security-constrained unit commitment (SCUC) model that considers the reserves provided by DR in the electricity markets, and demonstrated that utilizing DR as reserves significantly reduces operating costs and alleviates the congestion of transmission lines caused by grid component failures.

Recent advances in computing power and optimization algorithms have made it possible to incorporate the complicated characteristics of flexible loads and its integration in energy and reserve markets as DR providers. Works such as [26] consider both constant and voltage dependent loads in their market clearing co-optimization model, while [27] proposed a co-optimized market clearing model of energy and reserves, where DR is fully utilized by load-shifting bids in the energy market as well as spinning reserve bids in the reserve market. [28] proposed a day-ahead market clearing model that considers demand-side participation in the reserve market and a payback effect (load recovery) upon activation, and [29]-[30] analyzed the use of DR in the market clearing of energy and reserves considering wind generation and demand uncertainties. Furthermore, works such as [31] and [32] expanded the study of the topic beyond large-scale power systems, by analyzing the use of DR as an energy and reserve resource in micro-grid applications.

However, most of the aforementioned studies analyzed the benefits of DR under the ISOs perspective, where DR is mainly used as a tool to reduce system operating costs. Works that analyze the benefits of DR under this perspective tend to ignore the business model of DR providers and their opportunity costs when choosing to utilize their flexible loads to offer DR services as reserves instead of other alternatives, resulting in the overestimation of DR participation levels in the reserve market. This, coupled with ignoring market conditions of energy and reserve markets may distance previous studies from reality, since some of the main issues regarding low DR participation levels in real world applications is the uncertainty faced by DR providers when defining their business model, analyzing the profitability of different DR services and determining their potential revenue streams when offering these services to the market.

As a consequence, several works have recently analyzed the integration of DR in energy and reserve markets through the DR providers' perspective. In this set of studies, the benefits of DR are viewed under a portfolio management/business model rationale, where the main objective is to maximize DR providers' benefit (either via revenue maximization or cost reduction). Some of the most relevant works in this area include [33] and [34], which proposed operational models that enabled industrial loads to provide ancillary services (such as load following and regulation reserves) with the support of an energy storage system. Similarly, [33] also focused on industrial loads by proposing a DR model for a steel plant participating in the energy and spinning reserve markets, whereas [35] proposed a joint distributed generation (DG) and DR dispatch model for a virtual power player that operates the distribution network and participates in the energy and reserve markets. Besides DR participation in energy and reserve markets under the DR providers' perspective, other works have studied the integration of demand-side resources beyond DR. Such examples include [36] and [37], which proposed models to study the participation of an aggregator of prosumers in energy and reserve markets, and [38, 39, 40, 41] which proposed Electric Vehicle (EV) aggregator models that exploit EV charging capabilities to participate in the reserve market.

In addition to analyzing the integration of DR-based services in energy and reserve markets through the ISOs' perspective or through a portfolio management perspective, another key component in DR modeling research is how to accurately characterize load behavior and its expected consumption patterns in future operating scenarios. Due to the critical importance of load behavior when modeling DR-based services in electricity markets, several works have utilized different approaches to accurately represent how consumers react upon different types of DR programs.

One of the main approaches for load modeling relies on utilizing predefined parameters to represent consumer behavior. In this vein, works such as [42]-[43] utilize demand-side elasticities to derive loads' consumption levels and price sensitivities for their electricity consumption patterns. Authors in [27] instead utilize piece-wise functions to approximate a quadratic benefit function to represent DR providers' behavior. Some works such as [37] and [44] do not define a utility function to describe load behavior, but instead utilize predefined contracted volumes and load flexible capacities that are directly controlled by load aggregators. In addition to the aforementioned studies, other examples of authors utilizing predefined parameters to model load behavior can also be found on [45, 46, 47, 48].

However, utilizing predefined parameters for load characterization purposes can become highly sensitive on the assumptions and criteria behind the parameter selection process. In this set of studies, it is of critical importance to estimate in a realistic fashion the value of the parameters that characterize load behavior. Unfortunately, the lack of real-world applications of DR programs coupled with the availability problem of public data to generate realistic assumptions on parameters oftentimes makes the use of predefined parameters a challenging endeavor. Furthermore, as seen in real-world best practices of DR applications, different regions present different barriers for DR adoption, which generates replicability issues when using data from real-world DR programs from one specific region to estimate parameters for consumers located in other regions.

As a result, there is a growing interest in recent literature to deploy data-driven methods that minimize the use of predefined parameters to represent load behavior. In particular, inverse optimization (IO) has been attracting a great deal of attention in load modeling and DR research due to the ability to estimate consumption levels based on historical data, without the need to make assumptions about the nature of consumer behavior (such as defining price-demand elasticity, or predefined benefit functions) [49]. Moreover, this scheme is particularly useful in DR modeling, where IO can be used to estimate the price-response characteristics of consumers and use estimated parameters to elaborate pricing schemes [50] or efficient demand-side market bids [51].

In general terms, an IO scheme aims to estimate the value of the parameters of an optimization problem, given the solution of the respective decision variables. In these type of problems, estimated parameters could range from objective value coefficients, to right-hand and left-hand side parameters. Mathematically, the IO problem can be defined by considering the following linear optimization problem [52]:

$$\underset{x_j}{\text{Maximize}} \sum_{j \in \mathcal{J}} c_j x_j \tag{2.1}$$

subject to:

$$\sum_{j \in \mathcal{J}} a_{\mathrm{i}j} x_j \leq b_{\mathrm{i}}; \forall \mathrm{i} \in \mathcal{I}, \tag{2.2}$$

where set $\mathcal{J}$ corresponds to the index set of decision variable x, and set $\mathcal{I}$ denotes the index set of the constraints. Parameters b_{i}, c_j and $a_{\mathrm{i}j}$ correspond to right-hand side parameters, cost coefficients and constraint matrix coefficients, respectively.

Given an optimal solution $\hat{x}$ of problem (2.1)-(2.2), the IO problem seeks to infer parameters b_{i}, c_j and $a_{\mathrm{i}j}$ so that $\hat{x}$ is the solution of problem (2.1)-(2.2). This approach is particularly useful in DR modeling research, where load consumption levels can be utilized as observable solutions $\hat{x}$ of an underlying optimization problem, and IO is deployed to estimate the underlying parameters that characterize consumer behavior.

Originally proposed by [52], the IO problem initially focused on estimating cost coefficients c_j based on one observable sample of the decision variable $\hat{x}$. However, recent advances in optimization algorithms and computing power allow the IO problem to be further extended as a data-driven approach that can be utilized in cases where observable solutions of problem (2.1)-(2.2) are corrupted by noise [53], or in complex systems where the underlying parameters vary throughout the observable time horizon [54].

Due to its recent novelty in DR research, few works have applied IO-based schemes in their models. However, there are still some relevant contributions to the topic. In particular, authors in [50] utilize IO to estimate the price-response characteristics of consumers, which are later utilized in a one-leader multiple-follower Stackelberg game to determine the optimal pricing strategy for consumers to enable DR. Authors in [51] also focus in developing optimal pricing schemes, but instead deploy a tri-hierarchical strategy to represent the interaction between utility, load aggregators and consumers and utilize IO as a tool for utilities to estimate consumer power and cumulative energy boundaries. Works such as [55] instead utilize IO as a short-term forecasting tool for a pool of price-responsive loads, where this estimation method is used to estimate marginal utility curves and minimum and maximum power consumption limits. In addition to the aforementioned studies and its applications, other relevant works that deploy IO in a demand-side flexibility context can be found on [49, 56, 57].

Finally, and to the best of the authors' knowledge, despite several works utilizing IO-based schemes to forecast and characterize load behavior, no past studies have applied this approach under a portfolio management perspective, where the DR provider (retailer, load aggregator, etc.) utilizes IO for the final goal of offering multiple DR-based services to the system.

Chapter 3

A two-stage Modeling Framework for the Optimal Utilization of Load Flexibility in Energy and Reserve Markets

3.1 Two-stage Modeling Framework

3.1.1 Problem Statement and Modeling Assumptions

Problem Statement

Recent literature and best practices show that there is a growing interest in unlocking multiple DR-based services in electricity markets due to its multiple operational and reliability benefits to the system. However. there are several regions that present significant economic, regulatory and behavioral barriers that discourage retailers to adopt a multi-service approach. Under the perspective of these agents, revenue uncertainty coupled with the complexity of managing a flexible load portfolio that offers multiple services to the system may discourage them to adopt a multi-service strategy and instead solely utilize their flexible resources to reduce procurement costs in the energy market.

Hence, this work proposes a modeling framework that aims to optimize retailers' operations when offering multiple DR-based services to the system. In this framework, the retailer is modeled as a profit-maximizing entity that manages a portfolio of large-sized flexible loads. Each load has a predefined flexible load contract that allows the retailer to reduce its energy consumption upon activation. Therefore, the retailer can utilize these contracts individually to reduce procurement costs in the energy market, or aggregate them to place a bid in the reserve market.

Although the retailer has pre-established flexible load contracts with each load, it does not know a priori how these agents will behave in the future nor their real curtailment capabilities. As a consequence, the retailer must also characterize the future energy requirements and curtailment capabilities of its flexible load portfolio in order to determine the optimal utilization strategy between energy and reserve markets.

Modeling Assumptions

Our problem statement is influenced by several Latin American electricity markets such as Perú, Colombia and Chile. In these set of markets, retailers and large-sized (non-regulated) loads can freely establish long-term financial contracts between them [58, 59]. Based on this long-term nature of contracts between retailers and loads, we assume that flexible load contracts and curtailment obligations (contract price, curtailment windows, maximum curtailing capacity and maximum number of curtailment actions per year) are pre-established and do not change throughout the time horizon. Therefore, flexible loads are contractually bound to curtail their energy consumption when requested by the retailer and no bidirectional relationship exists between both agents.

In addition, we also assume that both energy and reserve markets are cleared independently, with the retailer being a price-taker in the former and a price-maker in the latter. This assumption is based on the market conditions of the Chilean Electricity market, where the wholesale energy market is highly liquid with multiple market participants, while the reserve market is characterized for having fewer participants and lower trading volumes [60]. Therefore, since traded volumes in the reserve market are significantly lower than in the energy market, both markets can be decoupled without major deviations from the real world context, and the retailer can be modeled a price-taker in the energy market and a price-maker in the reserve market.

Finally, and following the real world context of the Chilean Electricity market [60], we assume the reserve market to be auction-based, where market participants are remunerated using a pay-as-bid scheme.

3.1.2 Modeling Overview

In order to accurately characterize load behavior in future scenarios and determine the optimal utilization of retailers' flexible load contracts, the proposed modeling framework is divided in two stages. The first stage seeks to estimate the behavior of flexible loads managed by the retailer in future operating scenarios, while the second stage determines the optimal utilization of retailers' load flexibility in energy and reserve markets.

Fig. 3.1 depicts the proposed two-stage framework. The first stage aims to characterize demand profiles of large-sized loads for future operating scenarios. This characterization of demand consumption is carried out through IO [52], which aims to reconstruct the optimization problem solved by loads and estimate their parameters (namely, right-hand-side constraint parameters $b_{\rm i}$ and objective value coefficients c_j as depicted in problem (2.1)-(2.2)). The estimated parameters from these optimization problems - one per flexible load - are then inputted into the second stage, which aims to optimize retailers' operations. This second stage determines the optimal utilization of contracted flexible loads in day-ahead energy and reserve markets, where the model maximizes the profit of a retailer that leverages the flexibility of its load portfolio to offer DR-based services to the system.

For the first stage, the IO approach uses historical data (namely, past consumption and electricity prices) to estimate the unknown parameters of the optimization problem solved by large-sized loads when determining their daily energy requirements. This estimation process allows the creation of a model that represents load behavior and enables the prediction of future consumption levels. For each load, the IO scheme is formulated as a bilevel optimization problem, in which the lower level represents the optimization problem solved by loads, while the upper level seeks to estimate the parameters of the lower level such that prediction error is minimized [49].

For the second stage, the interaction between the retailer, the portfolio of flexible loads, and the reserve market is formulated using two distinct components. First, the retailer is modeled as a profit-maximizing entity that manages a portfolio of large-sized flexible loads. Since load behavior influences how the retailer utilizes its flexible load contracts, the optimization problem of each flexible load is integrated to constraint the optimization problem of the retailer. In addition, the parameters used to model flexible loads correspond to the ones estimated in the first stage through IO. The second component corresponds to the interaction between the retailer and the reserve market, which is formulated as a bilevel optimization problem. In this problem, the upper level corresponds to the decision-making process of the retailer, while the lower level represents the market clearing process of the reserve market.

Note that our model is designed from the retailers' perspective and thus it optimally utilizes its flexible load contracts to maximize its own profit. Additionally, the reserve market is modeled using an auction-based scheme, where the retailer has the capability to strategically decide the optimal price-quantity bid that maximizes the value of its flexible load contracts. Hence, the retailer has a direct influence on reserve clearing prices and total reserve costs perceived by the ISO.

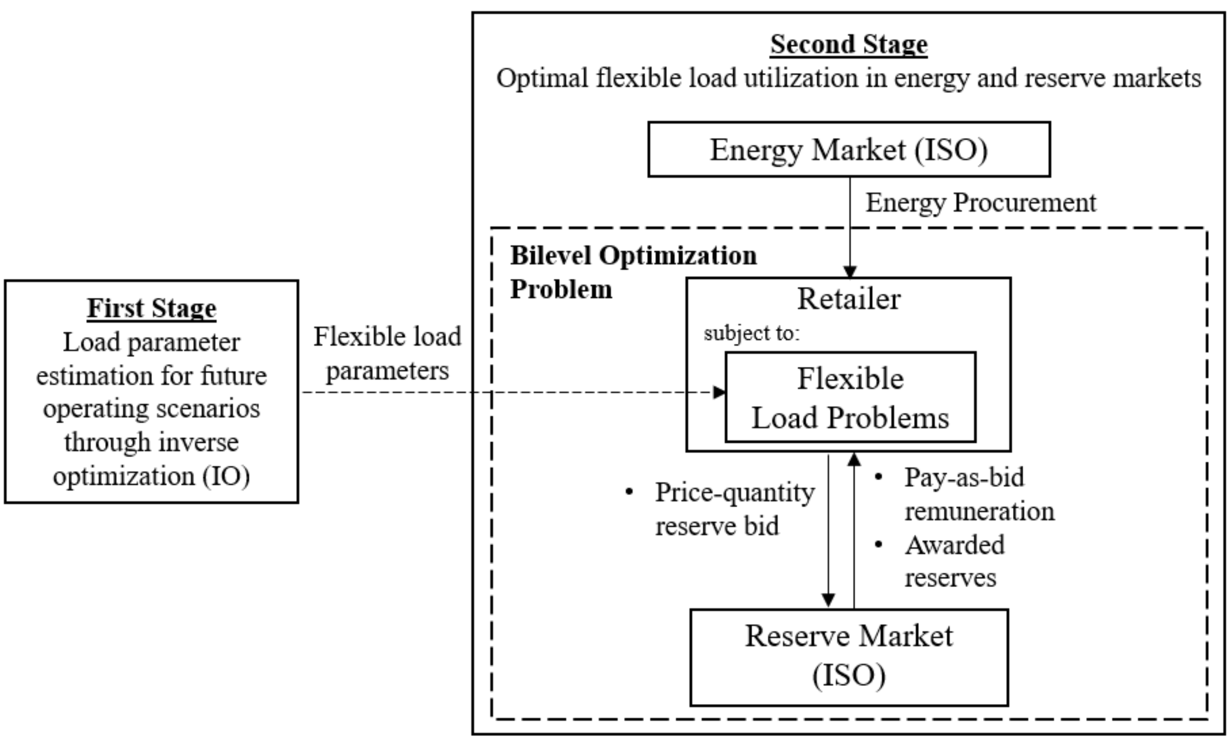

Figure 3.1: Diagram illustrating the proposed two-stage methodology

3.1.3 First Stage

As previously mentioned, the first step of this stage is to elaborate an optimization problem that can accurately represent the daily decision-making process carried out by each flexible load ($\forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}$) when determining their electricity consumption. In this context, and following the approach represented in problem (2.1)-(2.2), the proposed model is a linear optimization problem that is mathematically written as follows:

$$\underset{x_{\mathrm{cd}t}}{\text{Maximize}} \sum_{t \in T} x_{\mathrm{cd}t} \left(u_{\mathrm{cd}t} - \overline{\lambda}_{\mathrm{cd}t} \right) \tag{3.1}$$

subject to:

$$\overline{P}_{\mathrm{cd}t} \leq x_{\mathrm{cd}t} \leq \underline{P}_{\mathrm{cd}t} : (\overline{\mu}_{\mathrm{cd}t}, \underline{\mu}_{\mathrm{cd}t}); \forall t \in \mathcal{T} \tag{3.2}$$

$$x_{\mathrm{cd}t} - x_{\mathrm{cd}t-1} \leq r^{up}_{\mathrm{cd}t} : (\phi^{up}_{\mathrm{cd}t}); \forall t \in \mathcal{T} \setminus \{1\} \tag{3.3}$$

$$x_{\mathrm{cd}t-1} - x_{\mathrm{cd}t} \leq r^{\mathrm{down}}_{\mathrm{cd}t} : (\phi^{down}_{\mathrm{cd}t}); \forall t \in \mathcal{T} \setminus \{1\}, \tag{3.4}$$

where sets $\mathcal{C}, \mathcal{D}, \mathcal{T}$ include (in this order) indexes for the set of loads, days and hours of the day. The decision variable $x_{\mathrm{cd}t}$ corresponds to load energy consumption and parameters $u_{\mathrm{cd}t}$, $\overline{P}_{\mathrm{cd}t}$, $\underline{P}_{\mathrm{cd}t}$, $r^{up}_{\mathrm{cd}t}$, $r^{\mathrm{down}}_{\mathrm{cd}t}$, $\overline{\lambda}_{\mathrm{cd}t}$ correspond to marginal utility, maximum and minimum load consumption, maximum pick-up rate, minimum drop-off rate and preset contract price. Finally, dual variables $\overline{\mu}_{\mathrm{cd}t}$, $\underline{\mu}_{\mathrm{cd}t}$, $\phi^{up}_{\mathrm{cd}t}$ and $\phi^{down}_{\mathrm{cd}t}$ are included in parenthesis next to each constraint.

The objective function (3.1) represents loads' decision-making process for the day, which is driven by the difference between the marginal utility perceived for its energy consumption and the predefined contract price. Constraint (3.2) represents maximum and minimum consumption levels and constraints (3.3) and (3.4) represent the maximum pick-up and drop-off rates between two consecutive periods, respectively.

However, the set of parameters $\Phi^1_{\mathrm{cd}t} = \{u_{\mathrm{cd}t}, \overline{P}_{\mathrm{cd}t}, \underline{P}_{\mathrm{cd}t}, r^{up}_{\mathrm{cd}t}, r^{down}_{\mathrm{cd}t}\}$ is a priori unknown and may vary throughout different time periods ($t \in \mathcal{T}, \mathrm{d} \in \mathcal{D}$). In addition, each parameter may also depend on external variables such as consumption trends or market prices which in turn may potentially have non-linear relationships between them. Therefore, to accurately represent the dependence between parameters in set $\Phi^1_{\mathrm{cd}t}$ and external variables, Gaussian Kernel regression is utilized, which is formulated using the following methodology [56, 61]:

1. Define an array $z_{\mathrm{cd}t}$ comprised of all relevant external variables that may have an influence on parameters in set $\Phi^1_{\mathrm{cd}t}$ (such as previous consumption levels, market prices, previous weather conditions and other relevant metrics). As an example, the array $z_{\mathrm{cd}t}$ for a specific time period could be formulated using the last 3 days of electricity consumption at the same time of day (i.e. $z_{\mathrm{cd}t} = \{x_{c(\mathrm{d-i})t}\}$; $\forall \mathrm{i} \in \{1...3\}$). However, note that all external variables defined in array $z_{\mathrm{cd}t}$ must be known beforehand for each load and time horizon.
2. Select a set of training samples $\mathcal{S}$ based on historical data where load consumption $x_{\mathrm{cd}t}$ is known beforehand.

3. Define the Gaussian Kernel function for each time period based on the set of training samples $\mathcal{S}$:

$$K_{cs\mathrm{d}t} = \mathrm{e}^{-\gamma||z_{\mathrm{cd}t} - z_{cst}||_2^2}; \forall c \in \mathcal{C}, s \in \mathcal{S}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T}, \tag{3.5}$$

where coefficient γ models a scaling parameter used to determine the weight of the similarity measure between two arrays of external variables.

Once the Gaussian Kernel function K_{csdt} has been defined for each time period to account for the dependence with external variables, parameters in set $\Phi^1_{\mathrm{cd}t}$ can be mathematically described using Kernel regression as follows:

$$u_{\mathrm{cd}t} = u^0_{ct} + \sum_{s \in \mathcal{S}} \alpha_{1cst} K_{cs\mathrm{d}t}; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.6}$$

$$\underline{P}_{\mathrm{cd}t} = \underline{P}^0_{ct} + \sum_{s \in \mathcal{S}} \alpha_{2cst} K_{cs\mathrm{d}t}; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.7}$$

$$\overline{P}_{\mathrm{cd}t} = \overline{P}^0_{ct} + \sum_{s \in \mathcal{S}} \alpha_{3cst} K_{cs\mathrm{d}t}; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.8}$$

$$r^{up}_{\mathrm{cd}t} = r^{up0}_{ct} + \sum_{s \in \mathcal{S}} \alpha_{4cst} K_{cs\mathrm{d}t}; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.9}$$

$$r^{\mathrm{d}own}_{\mathrm{cd}t} = r^{\mathrm{d}own0}_{ct} + \sum_{s \in \mathcal{S}} \alpha_{5cst} K_{cs\mathrm{d}t}; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T}, \tag{3.10}$$

where parameters $\Phi^\alpha_{cst} = \{\alpha_{1cst}, \alpha_{2cst}, \alpha_{3cst}, \alpha_{4cst}, \alpha_{5cst}\}$ correspond to Kernel coefficients and parameters $\Phi^0_{ct} = \{u^0_{ct}, \underline{P}^0_{ct}, \overline{P}^0_{ct}, r^{up0}_{ct}, r^{\mathrm{d}own0}_{ct}\}$ correspond to intercepts.

Note that equations (3.6)-(3.10) show that unknown parameters of set $\Phi^1_{\mathrm{cd}t}$ can be mathematically expressed as a linear combination of Kernels derived from a training set $\mathcal{S}$, where consumption levels are known beforehand. Additionally, the Kernel function can be seen as a similarity measure between two arrays of external variables, in which a value between $(0, 1]$ is assigned based on their Euclidean distance [56].

Therefore, by selecting a training set of past consumption $\mathcal{S}$ and an array of external variables z_{cdt}, loads' full decision-making model for any time period is formulated as follows:

$$\underset{x_{\text{cdt}}}{\text{Maximize}} \sum_{t \in T} x_{\text{cdt}} \left(u^0_{ct} + \sum_{s \in \mathcal{S}} \alpha_{1cst} K_{cs\text{dt}} - \overline{\lambda}_{\text{cdt}} \right) \tag{3.11}$$

subject to:

$$\underline{P}^0_{ct} + \sum_{s \in \mathcal{S}} \alpha_{2cst} K_{cs\text{dt}} \leq x_{\text{cdt}} : (\underline{\mu}_{\text{cdt}}); \forall t \in \mathcal{T} \tag{3.12}$$

$$x_{\text{cdt}} \leq \overline{P}^0_{ct} + \sum_{s \in \mathcal{S}} \alpha_{3cst} K_{cs\text{dt}} : (\overline{\mu}_{\text{cdt}}); \forall t \in \mathcal{T} \tag{3.13}$$

$$x_{\text{cdt}} - x_{\text{cdt}-1} \leq r^{up0}_{ct} + \sum_{s \in \mathcal{S}} \alpha_{4cst} K_{cs\text{dt}} : (\phi^{up}_{\text{cdt}}); \quad \forall t \in \mathcal{T} \setminus \{1\} \tag{3.14}$$

$$x_{\text{cdt}-1} - x_{\text{cdt}} \leq r^{\text{down}0}_{ct} + \sum_{s \in \mathcal{S}} \alpha_{5cst} K_{cs\text{dt}} : (\phi^{\text{down}}_{\text{cdt}}); \quad \forall t \in \mathcal{T} \setminus \{1\}, \tag{3.15}$$

where Kernel coefficients $\Phi^{\alpha}_{cst} = \{\alpha_{1cst}, \alpha_{2cst}, \alpha_{3cst}, \alpha_{4cst}, \alpha_{5cst}\}$ and parameter intercepts $\Phi^0_{ct} = \{u^0_{ct}, \underline{P}^0_{ct}, \overline{P}^0_{ct}, r^{up0}_{ct}, r^{\text{down}0}_{ct}\}$ are a priori unknown and will be estimated using IO.

3.1.4 Second Stage

Once the first stage has been solved and parameters from model (3.11)-(3.15) have been estimated, the second stage of the modeling framework aims to determine the optimal utilization of flexible load contracts between energy and reserve markets. In this stage, the retailer is modeled as a profit-maximizing entity that manages a portfolio of large-sized loads, with each load having a predefined flexible load contract that allows the retailer to curtail energy consumption upon activation. Since the objective of the model is to determine the optimal utilization of flexible load contracts for future operating scenarios, flexible load behavior in this stage is modeled by incorporating the results from the first stage.

The retailer has two different ways to utilize its flexible load contracts and offer DR-based services to the system. First, the retailer can use its flexible load contracts in the energy market, where it can leverage loads' flexibility to reduce its energy consumption during periods of high energy prices. Second, the retailer has the option to aggregate its flexible load contracts and offer DR services as operating reserves in the reserve market. This market is modeled using an auction-based scheme, in which market participants submit price-quantity bids to provide reserves for the day-ahead. Once all bids have been submitted, the ISO clears the market by awarding operating reserve rights to the cheapest bids until reserve requirements are met, where winning offers to receive a remuneration based on a pay-as-bid scheme. In order to capture the interactions between the retailer, the wholesale markets and flexible loads, the model is formulated using the bilevel optimization scheme illustrated in Fig. 3.1.

As depicted in Fig. 3.1, the upper level corresponds to the retailers' model, where the optimization problem of each flexible load is integrated as additional constraints. Mathematically, the upper level is formulated as follows:

$$\underset{\substack{\Delta_{\text{cdt}}^{DR},\Delta_{\text{dt}}^{\text{bid}},\lambda_{\text{dt}}^{\text{bid}},\\ x_{\text{cdt}},\phi_{\text{cdt}}^{DR},\phi_{\text{cdt}}^{\text{bid}}}}{\text{Maximize}} \sum_{c\in\mathcal{C}}\sum_{\text{d}\in\mathcal{D}}\sum_{t\in\mathcal{T}} \Delta_{\text{cdt}}^{DR}\left(\lambda_{\text{dt}}^{E}-\overline{\lambda}_{\text{cdt}}\right) + \sum_{\text{d}\in\mathcal{D}}\sum_{t\in\mathcal{T}} \Delta_{\text{dt}}^{\text{bid},won}\lambda_{\text{dt}}^{\text{bid}} \tag{3.16}$$

subject to:

$$\Delta_{\text{dt}}^{\text{bid}} \leq \sum_{c\in\mathcal{C}} \overline{\Delta}_c\, \phi_{\text{cdt}}^{\text{bid}}; \forall \text{d}\in\mathcal{D}, t\in\mathcal{T} \tag{3.17}$$

$$\Delta_{\text{cdt}}^{DR} \leq \overline{\Delta}_c\, \phi_{\text{cdt}}^{DR}; \forall c\in\mathcal{C}, \text{d}\in\mathcal{D}, t\in\mathcal{T} \tag{3.18}$$

$$\phi_{\text{cdt}}^{DR} + \phi_{\text{cdt}}^{\text{bid}} \leq 1; \forall c\in\mathcal{C}, \text{d}\in\mathcal{D}, t\in\mathcal{T} \tag{3.19}$$

$$\sum_{t\in\mathcal{T}} \phi_{\text{cdt}}^{DR} + \sum_{t\in\mathcal{T}} \phi_{\text{cdt}}^{\text{bid}} \leq 1; \forall c\in\mathcal{C}, \text{d}\in\mathcal{D} \tag{3.20}$$

$$\sum_{\text{d}\in\mathcal{D}}\sum_{t\in\mathcal{T}} \phi_{\text{cdt}}^{DR} + \rho\sum_{\text{d}\in\mathcal{D}}\sum_{t\in\mathcal{T}} \phi_{\text{cdt}}^{\text{bid}} \leq \overline{\phi}_c; \forall c\in\mathcal{C} \tag{3.21}$$

$$x_{\text{cdt}} \in \Bigg\{ \underset{x_{\text{cdt}}}{\text{Maximize}} \sum_{t\in T} x_{\text{cdt}}\left(u_{\text{cdt}}-\overline{\lambda}_{\text{cdt}}\right) \tag{3.22}$$

subject to:

$$\underline{P}_{\text{cdt}} \leq x_{\text{cdt}} : (\underline{\mu}_{\text{cdt}}); \forall t\in\mathcal{T} \tag{3.23}$$

$$x_{\text{cdt}} \leq \overline{P}_{\text{cdt}} - \Delta_{\text{cdt}}^{DR} : (\overline{\mu}_{\text{cdt}}); \forall t\in\mathcal{T} \tag{3.24}$$

$$x_{\text{cdt}} - x_{\text{cdt}-1} \leq r_{\text{cdt}}^{up} : (\phi_{\text{cdt}}^{up}); \forall t\in\mathcal{T}\setminus\{1\} \tag{3.25}$$

$$x_{\text{cdt}-1} - x_{\text{cdt}} \leq r_{\text{cdt}}^{\text{d}own} : (\phi_{\text{cdt}}^{\text{d}own}); \forall t\in\mathcal{T}\setminus\{1\}\Bigg\}, \forall c\in\mathcal{C}, \text{d}\in\mathcal{D}, \tag{3.26}$$

where the decision variables are DR volume provided in the energy market, Δ_{cdt}^{DR}, bid size in the reserve market, $\Delta_{\text{dt}}^{\text{bid}}$, bidding price, $\lambda_{\text{dt}}^{\text{bid}}$, load energy consumption, x_{cdt}, call to use flexible load contract as DR provision in the energy market, ϕ_{cdt}^{DR}, call to use flexible contract to bid in the reserve market, $\phi_{\text{cdt}}^{\text{bid}}$. Parameters $\overline{\Delta}_c$ and $\overline{\phi}_c$ represent contracted flexible load and maximum number of times that the flexible load contract can be utilized in a time horizon. Finally, objective function parameters λ_{dt}^{E} and $\overline{\lambda}_{\text{cdt}}$ represent energy price and preset contract price of flexible loads.

Regarding the optimization problem of flexible loads, parameters from the set $\Phi_{\text{cdt}}^{1} = \{u_{\text{cdt}}, \overline{P}_{\text{cdt}}, \underline{P}_{\text{cdt}}, r_{\text{cdt}}^{up}, r_{\text{cdt}}^{\text{d}own}\}$ correspond to the recovered load parameters derived from the first stage (and mathematically formulated using Gaussian Kernel regression based on equations (3.6)-(3.9)). The objective function and constraints (3.22)-(3.26) represent flexible loads' model, which is analogous to one used in the first stage with the exception that the parameter Δ_{cdt}^{DR} is incorporated in equation (3.24) to impose an upper bound to loads' energy consumption upon activation by the retailer.

The objective function (3.16) to be maximized includes the two revenue streams derived from using flexible load contracts, namely, savings derived from utilizing these contracts to reduce loads' consumption in the energy market (during time periods of high energy prices) and the pay-as-bid remuneration of the reserve market. Constraint (3.17) imposes limits to the amount bid in the reserve market, constraint (3.18) limits DR provision in the energy market, constraint (3.19) imposes that the flexible load contract can be only utilized in one market at a time (energy or reserve), and constraints (3.20)-(3.21) limit the utilization of flexible load contracts in a day and throughout the time horizon respectively.

Note that coefficient ρ is incorporated into constraint (3.21) to represent the activation rate of reserves, where a value between $(0, 1]$ is used to represent the number of bids that can be made before utilizing the flexible load contract in the reserve market (due to reserve deployment during real-time operation).

The lower level problem represents the market-clearing process carried out by the retailer in the reserve market and is formulated as follows:

$$\underset{\substack{P_{gdt}^{\text{bid},won}, \\ \Delta_{\text{dt}}^{\text{bid},won}}}{\text{Minimize}} \sum_{t\in T}\sum_{g\in G} P_{g\text{dt}}^{\text{bid},won} C_{g\text{dt}}^{gen} + \sum_{t\in T} \Delta_{\text{dt}}^{\text{bid},won} \lambda_{\text{dt}}^{\text{bid}} \tag{3.27}$$

subject to:

$$\sum_{g\in G} P_{g\text{dt}}^{\text{bid},won} + \Delta_{\text{dt}}^{\text{bid},won} = R_t; (\eta_{\text{dt}}); \forall t \in \mathcal{T} \tag{3.28}$$

$$0 \leq P_{g\text{dt}}^{\text{bid},won} \leq \overline{P}_{g\text{dt}}^{\text{bid}} : (\underline{\kappa}_{g\text{dt}}, \overline{\kappa}_{g\text{dt}}); \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{3.29}$$

$$0 \leq \Delta_{\text{dt}}^{\text{bid},won} \leq \Delta_{\text{dt}}^{\text{bid}} : (\underline{\delta}_{\text{dt}}, \overline{\delta}_{\text{dt}}); \forall t \in \mathcal{T}, \forall \text{d} \in \mathcal{D}, \tag{3.30}$$

where $\mathcal{G}$ is the set of indexes of generators bidding in the reserve market. The decision variable of this sub-problem are the amount of reserves won by the retailer in the reserve market, $\Delta_{\text{dt}}^{\text{bid},won}$ and the amount won by generators in the reserve market, $P_{g\text{dt}}^{\text{bid},won}$. Parameters $\overline{P}_{g\text{dt}}^{\text{bid}}$ and $C_{g\text{dt}}^{gen}$ represent bid size and price made by participating generators in the reserve market, parameter R_t represents reserve requirements, and dual variables are added in parenthesis next to each constraint. The objective function (3.27) to be minimized includes bids made by participating generators and the retailer, while constraint (3.28) models reserve balance and constraints (3.29)-(3.30) limit the maximum amount to be awarded to each participating agent based on their market bid.

By integrating both components of the second stage, the full model is mathematically formulated as follows:

$$\underset{\substack{\Delta_{\text{cdt}}^{DR}, \Delta_{\text{dt}}^{\text{bid}}, \lambda_{\text{dt}}^{\text{bid}}, \\ x_{\text{cdt}}, \phi_{\text{cdt}}^{DR}, \phi_{\text{cdt}}^{\text{bid}}}}{\text{Maximize}} \sum_{c\in\mathcal{C}}\sum_{\text{d}\in\mathcal{D}}\sum_{t\in\mathcal{T}} \Delta_{\text{cdt}}^{DR}\left(\lambda_{\text{dt}}^{E} - \overline{\lambda}_{\text{cdt}}\right)$$
$$+ \sum_{\text{d}\in\mathcal{D}}\sum_{t\in\mathcal{T}} \Delta_{\text{dt}}^{\text{bid},won} \lambda_{\text{dt}}^{\text{bid}} \tag{3.31}$$

subject to:

$$\Delta_{\mathrm{d}t}^{\mathrm{bid}} \leq \sum_{c \in \mathcal{C}} \overline{\Delta}_c \, \phi_{\mathrm{cd}t}^{\mathrm{bid}}; \forall \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.32}$$

$$\Delta_{\mathrm{cd}t}^{DR} \leq \overline{\Delta}_c \, \phi_{\mathrm{cd}t}^{DR}; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.33}$$

$$\phi_{\mathrm{cd}t}^{DR} + \phi_{\mathrm{cd}t}^{\mathrm{bid}} \leq 1; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.34}$$

$$\sum_{t \in \mathcal{T}} \phi_{\mathrm{cd}t}^{DR} + \sum_{t \in \mathcal{T}} \phi_{\mathrm{cd}t}^{\mathrm{bid}} \leq 1; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D} \tag{3.35}$$

$$\sum_{\mathrm{d} \in \mathcal{D}} \sum_{t \in \mathcal{T}} \phi_{\mathrm{cd}t}^{DR} + \rho \sum_{\mathrm{d} \in \mathcal{D}} \sum_{t \in \mathcal{T}} \phi_{\mathrm{cd}t}^{\mathrm{bid}} \leq \overline{\phi}_c; \forall c \in \mathcal{C} \tag{3.36}$$

$$x_{\mathrm{cd}t} \in \Big\{ \underset{x_{\mathrm{cd}t}}{\text{Maximize}} \sum_{t \in T} x_{\mathrm{cd}t} \left(u_{\mathrm{cd}t} - \overline{\lambda}_{\mathrm{cd}t} \right) \tag{3.37}$$

subject to:

$$\underline{P}_{\mathrm{cd}t} \leq x_{\mathrm{cd}t} : (\underline{\mu}_{\mathrm{cd}t}); \forall t \in \mathcal{T} \tag{3.38}$$

$$x_{\mathrm{cd}t} \leq \overline{P}_{\mathrm{cd}t} - \Delta_{\mathrm{cd}t}^{DR} : (\overline{\mu}_{\mathrm{cd}t}); \forall t \in \mathcal{T} \tag{3.39}$$

$$x_{\mathrm{cd}t} - x_{\mathrm{cd}t-1} \leq r_{\mathrm{cd}t}^{up} : (\phi_{\mathrm{cd}t}^{up}); \forall t \in \mathcal{T} \setminus \{1\} \tag{3.40}$$

$$x_{\mathrm{cd}t-1} - x_{\mathrm{cd}t} \leq r_{\mathrm{cd}t}^{\mathrm{down}} : (\phi_{\mathrm{cd}t}^{\mathrm{down}}); \quad \forall t \in \mathcal{T} \setminus \{1\} \Big\}, \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D} \tag{3.41}$$

$$\Delta_{\mathrm{d}t}^{\mathrm{bid},won} \in \Big\{ \underset{\substack{P_{g\mathrm{d}t}^{\mathrm{bid},won}, \\ \Delta_{\mathrm{d}t}^{\mathrm{bid},won}}}{\text{Minimize}} \sum_{t \in T} \sum_{g \in G} P_{g\mathrm{d}t}^{\mathrm{bid},won} C_{g\mathrm{d}t}^{gen} + \sum_{t \in T} \Delta_{\mathrm{d}t}^{\mathrm{bid},won} \lambda_{\mathrm{d}t}^{\mathrm{bid}} \tag{3.42}$$

subject to:

$$\sum_{g \in G} P_{g\mathrm{d}t}^{\mathrm{bid},won} + \Delta_{\mathrm{d}t}^{\mathrm{bid},won} = R_t : (\eta_{\mathrm{d}t}); \quad \forall t \in \mathcal{T} \tag{3.43}$$

$$0 \leq P_{g\mathrm{d}t}^{\mathrm{bid},won} \leq \overline{P}_{g\mathrm{d}t}^{\mathrm{bid}} : (\underline{\kappa}_{g\mathrm{d}t}, \overline{\kappa}_{g\mathrm{d}t}); \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{3.44}$$

$$0 \leq \Delta_{\mathrm{d}t}^{\mathrm{bid},won} \leq \Delta_{\mathrm{d}t}^{\mathrm{bid}} : (\underline{\delta}_{\mathrm{d}t}, \overline{\delta}_{\mathrm{d}t}); \forall t \in \mathcal{T} \Big\}, \quad \forall \mathrm{d} \in \mathcal{D} \tag{3.45}$$

$$\phi_{\mathrm{cd}t}^{\mathrm{bid}}, \phi_{\mathrm{cd}t}^{DR} \in \{0, 1\}; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.46}$$

3.2 Solution Methodology

3.2.1 First Stage

Equations (3.11)-(3.15) mathematically describe the decision-making process of loads' daily energy consumption, where Kernel regression is utilized to model the relationship with external variables. However, the retailer does not know a priori the value of said parameters, (namely set Φ^1_{cdt} which is represented through Kernel regression). Hence, due to the linear nature of the proposed model combined with the potential availability of historical data, the estimation of unknown parameters can be done using inverse optimization (IO) [52].

Given a training set $\mathcal{S}$ containing samples of past consumption $\hat{x}_{cst}$, electricity prices $\overline{\lambda}_{cst}$ and an array of external variables z_{cst}, this methodology seeks to estimate parameters in set Φ^1_{cst}, such that the difference between the estimated consumption derived from the model x_{cst} and real consumption $\hat{x}_{cst}$ is minimized. In other words, the IO methodology aims to estimate the underlying parameters of loads' decision-making model that best fits the observable consumption levels of the time periods within the training set $\mathcal{S}$.

Mathematically, the IO model for each load ($\forall c \in \mathcal{C}$) using a set of samples $\mathcal{S}$ is formulated as the following bilevel program:

$$\underset{\mathbf{x}_{cst},\Phi^1_{cst}}{\text{Minimize}} \ \frac{1}{|S|}\sum_{s\in\mathcal{S}}\sum_{t\in T}|x_{cst}-\hat{x}_{cst}| \tag{3.47}$$

$$x_{cst} \in \Big\{ \underset{x_{cst}}{\text{Maximize}} \sum_{t\in T} x_{cst}\left(u_{cst}-\overline{\lambda}_{cst}\right) \tag{3.48}$$

$$\text{subject to:}$$

$$\underline{P}_{cst} \leq x_{cst} : (\underline{\mu}_{cst}); \forall t \in \mathcal{T} \tag{3.49}$$

$$x_{cst} \leq \overline{P}_{cst} : (\overline{\mu}_{cst}); \forall t \in \mathcal{T} \tag{3.50}$$

$$x_{cst} - x_{cst-1} \leq r^{up}_{cst} : (\phi^{up}_{cst}); \forall t \in \mathcal{T} \setminus \{1\} \tag{3.51}$$

$$x_{cst-1} - x_{cst} \leq r^{\text{down}}_{cst} : (\phi^{\text{down}}_{cst}); \quad \forall t \in \mathcal{T} \setminus \{1\} \Big\}, \forall s \in \mathcal{S}, \tag{3.52}$$

where parameters from set $\Phi^1_{cst} = \{u_{cst}, \overline{P}_{cst}, \underline{P}_{cst}, r^{up}_{cst}, r^{\text{down}}_{cst}\}$ are mathematically formulated using the Gaussian Kernel regression formula:

$$u_{cst} = u^0_{ct} + \sum_{p\in\mathcal{S}} \alpha_{1cpt} K_{cpst}; \forall c \in \mathcal{C}, s \in \mathcal{S}, \quad t \in \mathcal{T} \tag{3.53}$$

$$\underline{P}_{cst} = \underline{P}^0_{ct} + \sum_{p\in\mathcal{S}} \alpha_{2cpt} K_{cpst}; \forall c \in \mathcal{C}, s \in \mathcal{S},$$

$$t \in \mathcal{T} \tag{3.54}$$

$$\overline{P}_{cst} = \overline{P}^0_{ct} + \sum_{p \in \mathcal{S}} \alpha_{3cpt} K_{cpst}; \forall c \in \mathcal{C}, s \in \mathcal{S}, \quad t \in \mathcal{T} \tag{3.55}$$

$$r^{up}_{cst} = r^{up0}_{ct} + \sum_{p \in \mathcal{S}} \alpha_{4cpt} K_{cpst}; \forall c \in \mathcal{C}, s \in \mathcal{S}, \quad t \in \mathcal{T} \tag{3.56}$$

$$r^{\text{down}}_{cst} = r^{down0}_{ct} + \sum_{p \in \mathcal{S}} \alpha_{5cpt} K_{cpst}; \forall c \in \mathcal{C}, \quad s \in \mathcal{S}, t \in \mathcal{T}, \tag{3.57}$$

with p being a sub-index used to denote other time periods from the training sample $\mathcal{S}$. Kernel regression parameters are represented by sets $\Phi^{\alpha}_{cpt} = \{\alpha_{1cpt}, \alpha_{2cpt}, \alpha_{3cpt}, \alpha_{4cpt}, \alpha_{5cpt}\}$ and $\Phi^0_{ct} = \{u^0_{ct}, \underline{P}^0_{ct}, \overline{P}^0_{ct}, r^{up0}_{ct}, r^{down0}_{ct}\}$.

Using KKT conditions, model (3.47)-(3.52) can be rewritten as:

$$\underset{\substack{\zeta_{1cst}, \zeta_{2cst}, x_{cst}, \\ \Theta_{cst}, \Phi^1_{cst}, \Phi^0_{ct}, \Phi^{\alpha}_{cpt}}}{\text{Minimize}} \quad \frac{1}{|S|} \sum_{s \in \mathcal{S}} \sum_{t \in T} (\zeta_{1cst} + \zeta_{2cst}) \tag{3.58}$$

subject to:

$$x_{cst} - \hat{x}_{cst} = \zeta_{1,cst} + \zeta_{2,cst}; \forall s \in \mathcal{S}, t \in \mathcal{T} \tag{3.59}$$

$$\underline{P}_{cst} \le x_{cst}; \forall s \in \mathcal{S}, t \in \mathcal{T} \tag{3.60}$$

$$x_{cst} \le \overline{P}_{cst}; \forall s \in \mathcal{S}, t \in \mathcal{T} \tag{3.61}$$

$$x_{cst} - x_{cst-1} \le r^{up}_{cst}; \forall s \in \mathcal{S}, t \in \mathcal{T} \setminus \{1\} \tag{3.62}$$

$$x_{cst-1} - x_{cst} \le r^{\text{down}}_{cst}; \forall s \in \mathcal{S}, t \in \mathcal{T} \setminus \{1\} \tag{3.63}$$

$$\phi^{up}_{cst+1} - \phi^{\text{down}}_{cst+1} + \underline{\mu}_{cst} - \overline{\mu}_{cst} + u_{cst} - \overline{\lambda}_{cst} = 0; \quad \forall s \in \mathcal{S}, t = 1 \tag{3.64}$$

$$\phi^{up}_{cst+1} - \phi^{up}_{cst} - \phi^{\text{down}}_{cst+1} + \phi^{\text{down}}_{cst} + \underline{\mu}_{cst} - \overline{\mu}_{cst} + u_{cst} - \overline{\lambda}_{cst} = 0; \forall s \in \mathcal{S}, t \in \mathcal{T} \setminus \{1, T\} \tag{3.65}$$

$$-\phi^{up}_{cst} + \phi^{\text{down}}_{cst} + \underline{\mu}_{cst} - \overline{\mu}_{cst} + u_{cst} - \overline{\lambda}_{cst} = 0; \quad \forall s \in \mathcal{S}, t = T \tag{3.66}$$

$$0 \le \underline{\mu}_{cst} \perp 0 \le x_{cst} - \underline{P}_{cst}; \forall s \in \mathcal{S}, t \in \mathcal{T} \tag{3.67}$$

$$0 \le \overline{\mu}_{cst} \perp 0 \le \overline{P}_{cst} - x_{cst}; \forall s \in \mathcal{S}, t \in \mathcal{T} \tag{3.68}$$

$$0 \le \phi^{up}_{cst} \perp 0 \le r^{up}_{cst} - x_{cst} + x_{cst-1}; \forall s \in \mathcal{S}, \quad t \in \mathcal{T} \setminus \{1\} \tag{3.69}$$

$$0 \le \phi^{down}_{cst} \perp 0 \le r^{\text{down}}_{cst} - x_{cst-1} + x_{cst}; \forall s \in \mathcal{S}, \quad t \in \mathcal{T} \setminus \{1\} \tag{3.70}$$

$$0 \le \zeta_{1cst}, \zeta_{2cst}; \forall s \in \mathcal{S}, t \in \mathcal{T}, \tag{3.71}$$

where set $\Theta_{cst} = \{\overline{\mu}_{cst}, \underline{\mu}_{cst} \phi^{up}_{cst}\, \phi^{\text{down}}_{cst}\}$ corresponds to the set of dual variables associated with constraints (3.49)-(3.52) and variables ζ_{1cst}, ζ_{2cst} are used to remove the absolute value of objective function (3.47).

Once model (3.47)-(3.52) has been reformulated using KKT conditions, complementarity constraints (3.67)-(3.70) can be relaxed using the penalty method based on Schur's decomposition proposed in [62]:

$$\underset{\substack{\zeta_{1cst},\zeta_{2cst},x_{cst},\\ \Theta_{cst},\Phi^1_{cst},\Phi^0_{ct},\Phi^{\alpha}_{cpt},\\ v^{up}_{\text{ist}},v^{\text{down}}_{\text{ist}}}}{\text{Minimize}} \quad \frac{1}{|S|}\Big[\sum_{s\in\mathcal{S}}\sum_{t\in T}(\zeta_{1cst}+\zeta_{2cst}) + \sum_{\text{i}\in\mathcal{I}}\sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}} L_{\text{ist}}\left(v^{up}_{\text{ist}}+v^{\text{down}}_{\text{ist}}\right)\Big] \tag{3.72}$$

subject to:

$$x_{cst} - \hat{x}_{cst} = \zeta_{1,cst} + \zeta_{2,cst}; \forall s \in \mathcal{S}, t \in \mathcal{T} \tag{3.73}$$

$$\underline{P}_{cst} \leq x_{cst}; \forall s \in \mathcal{S}, t \in \mathcal{T} \tag{3.74}$$

$$x_{cst} \leq \overline{P}_{cst}; \forall s \in \mathcal{S}, t \in \mathcal{T} \tag{3.75}$$

$$x_{cst} - x_{cst-1} \leq r^{up}_{cst}; \forall s \in \mathcal{S}, t \in \mathcal{T}\setminus\{1\} \tag{3.76}$$

$$x_{cst-1} - x_{cst} \leq r^{\text{down}}_{cst}; \forall s \in \mathcal{S}, t \in \mathcal{T}\setminus\{1\} \tag{3.77}$$

$$\phi^{up}_{cst+1} - \phi^{\text{down}}_{cst+1} + \underline{\mu}_{cst} - \overline{\mu}_{cst} + u_{cst} - \overline{\lambda}_{cst} = 0; \quad \forall s \in \mathcal{S}, t = 1 \tag{3.78}$$

$$\phi^{up}_{cst+1} - \phi^{up}_{cst} - \phi^{\text{down}}_{cst+1} + \phi^{\text{down}}_{cst} + \underline{\mu}_{cst} - \overline{\mu}_{cst} + u_{cst} - \overline{\lambda}_{cst} = 0; \forall s \in \mathcal{S}, t \in \mathcal{T}\setminus\{1, T\} \tag{3.79}$$

$$-\phi^{up}_{cst} + \phi^{\text{down}}_{cst} + \underline{\mu}_{cst} - \overline{\mu}_{cst} + u_{cst} - \overline{\lambda}_{cst} = 0; \quad \forall s \in \mathcal{S}, t = T \tag{3.80}$$

$$0.5\left(\underline{\mu}_{cst} + x_{cst} - \underline{P}_{cst}\right) = v^{up}_{1st} - v^{down}_{1st}; \forall s \in \mathcal{S}, t \in \mathcal{T} \tag{3.81}$$

$$0.5\left(\overline{\mu}_{cst} + \overline{P}_{cst} - x_{cst}\right) = v^{up}_{2st} - v^{\text{down}}_{2st}; \forall s \in \mathcal{S}, t \in \mathcal{T} \tag{3.82}$$

$$0.5\,(\phi^{up}_{cst} + r^{up}_{cst} - x_{cst} + x_{cst-1}) = v^{up}_{3st} - v^{\text{down}}_{3st}; \quad \forall s \in \mathcal{S}, t \in \mathcal{T}\setminus\{1\} \tag{3.83}$$

$$0.5\left(\phi^{\text{down}}_{cst} + r^{\text{down}}_{cst} - x_{cst-1} + x_{cst}\right) = v^{up}_{4st} - v^{\text{down}}_{4st}; \quad \forall s \in \mathcal{S}, t \in \mathcal{T}\setminus\{1\} \tag{3.84}$$

$$0.5\left(\underline{\mu}_{cst} - x_{cst} + \underline{P}_{cst}\right) = v^{up}_{1st} - v^{down}_{1st}; \forall s \in \mathcal{S}, t \in \mathcal{T} \tag{3.85}$$

$$0.5\left(\overline{\mu}_{cst} - \overline{P}_{cst} + x_{cst}\right) = v^{up}_{2st} - v^{\text{down}}_{2st}; \forall s \in \mathcal{S}, t \in \mathcal{T} \tag{3.86}$$

$$0.5\left(\phi^{up}_{cst} - r^{up}_{cst} + x_{cst} - x_{cst-1}\right) = v^{up}_{3st} - v^{\text{down}}_{3st};$$
$$\forall s \in \mathcal{S}, t \in \mathcal{T} \setminus \{1\} \quad (3.87)$$
$$0.5\left(\phi^{\text{down}}_{cst} - r^{\text{down}}_{cst} + x_{cst-1} - x_{cst}\right) = v^{up}_{4st} - v^{\text{down}}_{4st};$$
$$\forall s \in \mathcal{S}, t \in \mathcal{T} \setminus \{1\} \quad (3.88)$$
$$0 \leq v^{up}_{\text{i}st}, v^{\text{down}}_{\text{i}st}; \forall \text{i} \in \mathcal{I} \setminus \{3,4\}, s \in \mathcal{S}, t \in \mathcal{T} \quad (3.89)$$
$$0 \leq v^{up}_{\text{i}st}, v^{\text{down}}_{\text{i}st}; \forall \text{i} \in \mathcal{I} \setminus \{1,2\}, s \in \mathcal{S}, t \in \mathcal{T} \setminus \{1\} \quad (3.90)$$
$$0 \leq \zeta_{1cst}, \zeta_{2cst}; \forall s \in \mathcal{S}, t \in \mathcal{T}, \quad (3.91)$$

where $\mathcal{I} = \{1...4\}$ is the set of indexes of penalty variables $v^{up}_{\text{i}st}$, $v^{\text{down}}_{\text{i}st}$ and $L_{\text{i}st}$ corresponds to the penalty parameter derived from Schur's decomposition.

Note that, in a similar fashion to previous works (such as [49] and [50]), the original objective function (3.47) is designed to minimize in-sample prediction error. In fact, as noted in [50], solving (3.47)-(3.52) to optimality does not imply that estimated parameters correspond to the ones that generate the minimal out-of-sample error. In the case presented in this work, this phenomena is of significant importance due to the fact that estimated parameters are assumed to have a direct relationship with external factors that present significant variations throughout the operating horizon (such as electricity prices, weather conditions, etc.), which may in turn cause that the parameters estimated from solving (3.47)-(3.52) to optimality may not accurately represent future load behavior.

Therefore, since the objective of this stage is to accurately estimate parameters for out-of-sample operating scenarios, the proposed methodology does not solve the model to optimality, but instead leverages the tuning of the penalty parameter $L_{\text{i}st}$ to minimize out-of-sample prediction error. In this vein, several testing algorithms such as cross-validation and grid search techniques can be utilized to accurately tune $L_{\text{i}st}$ so that estimated parameters correspond to the ones that minimize out-of-sample prediction error. Hence, by not solving model (3.47)-(3.52) to optimality and instead leveraging the proposed decomposition and the tuning of the penalty parameter $L_{\text{i}st}$ to minimize out-of-sample error, the model is capable of first, reducing computational costs, and second, provide more accurate results regarding prediction capabilities.

Once penalty parameter $L_{\text{i}st}$ has been tuned to minimize out-of-sample error, and model (3.72)-(3.91) has been solved for a training sample $\mathcal{S}$, recovered parameters from sets Φ^0_{ct} and Φ^{α}_{cpt} can be used to estimate out-of-sample consumption behavior (i.e. $\forall c \in \mathcal{C}, \text{d} \in \mathcal{D}, t \in \mathcal{T}$). To do so, if the array of external variables $z_{c\text{d}t}$ is known for an out-of-sample period, the Gaussian Kernel function $K_{cs\text{d}t}$ can be determined for said period (see equation (3.5)) and recovered parameters can be imputed in model (3.11)-(3.15) to fully characterize the optimization problem solved by loads when determining their out-of-sample electricity consumption.

3.2.2 Second Stage

The two-component formulation shown in Fig. 3.1 and mathematically represented in (3.31)-(3.46) can be reformulated using KKT conditions:

$$\underset{\substack{\Delta^{DR}_{\mathrm{cdt}},\lambda^{\mathrm{bid}}_{\mathrm{dt}},\Delta^{\mathrm{bid},won}_{\mathrm{dt}},\\ x_{\mathrm{cdt}},\phi^{DR}_{\mathrm{cdt}},\phi^{\mathrm{bid}}_{\mathrm{cdt}},\Delta^{\mathrm{bid}}_{\mathrm{dt}},\\ \Omega^1_{\mathrm{dt}},\Omega^2_{g\mathrm{dt}},\Omega^3_{\mathrm{cdt}}}}{\text{Maximize}} \sum_{c\in\mathcal{C}}\sum_{\mathrm{d}\in\mathcal{D}}\sum_{t\in\mathcal{T}} \Delta^{DR}_{\mathrm{cdt}}\left(\lambda^{E}_{\mathrm{dt}} - \overline{\lambda}_{\mathrm{cdt}}\right) + \sum_{\mathrm{d}\in\mathcal{D}}\sum_{t\in\mathcal{T}} \Delta^{\mathrm{bid},won}_{\mathrm{dt}}\lambda^{\mathrm{bid}}_{\mathrm{dt}} \tag{3.92}$$

subject to:

$$\Delta^{\mathrm{bid}}_{\mathrm{dt}} \le \sum_{c\in\mathcal{C}} \overline{\Delta}_c\, \phi^{\mathrm{bid}}_{\mathrm{cdt}}; \forall \mathrm{d}\in\mathcal{D}, t\in\mathcal{T} \tag{3.93}$$

$$\Delta^{DR}_{\mathrm{cdt}} \le \overline{\Delta}_c\, \phi^{DR}_{\mathrm{cdt}}; \forall c\in\mathcal{C}, \mathrm{d}\in\mathcal{D}, t\in\mathcal{T} \tag{3.94}$$

$$\phi^{DR}_{\mathrm{cdt}} + \phi^{\mathrm{bid}}_{\mathrm{cdt}} \le 1; \forall c\in\mathcal{C}, \mathrm{d}\in\mathcal{D}, t\in\mathcal{T} \tag{3.95}$$

$$\sum_{t\in\mathcal{T}}\phi^{DR}_{\mathrm{cdt}} + \sum_{t\in\mathcal{T}}\phi^{\mathrm{bid}}_{\mathrm{cdt}} \le 1; \forall c\in\mathcal{C}, \mathrm{d}\in\mathcal{D} \tag{3.96}$$

$$\sum_{\mathrm{d}\in\mathcal{D}}\sum_{t\in\mathcal{T}}\phi^{DR}_{\mathrm{cdt}} + \rho\sum_{\mathrm{d}\in\mathcal{D}}\sum_{t\in\mathcal{T}}\phi^{\mathrm{bid}}_{\mathrm{cdt}} \le \overline{\phi}_c; \forall c\in\mathcal{C} \tag{3.97}$$

$$\phi^{up}_{\mathrm{cd}t+1} - \phi^{\mathrm{d}own}_{\mathrm{cd}t+1} + \underline{\mu}_{\mathrm{cdt}} - \overline{\mu}_{\mathrm{cdt}} + u_{\mathrm{cdt}} - \overline{\lambda}_{\mathrm{cdt}} = 0; \quad \forall c\in\mathcal{C}, \mathrm{d}\in\mathcal{D}, t=1 \tag{3.98}$$

$$\phi^{up}_{\mathrm{cd}t+1} - \phi^{up}_{\mathrm{cdt}} - \phi^{\mathrm{d}own}_{\mathrm{cd}t+1} + \phi^{\mathrm{d}own}_{\mathrm{cdt}} + \underline{\mu}_{\mathrm{cdt}} - \overline{\mu}_{\mathrm{cdt}} + u_{\mathrm{cdt}} - \overline{\lambda}_{\mathrm{cdt}} = 0; \forall c\in\mathcal{C}, \mathrm{d}\in\mathcal{D}, t\in\mathcal{T}\setminus\{1,T\} \tag{3.99}$$

$$-\phi^{up}_{\mathrm{cdt}} + \phi^{\mathrm{d}own}_{\mathrm{cdt}} + \underline{\mu}_{\mathrm{cdt}} - \overline{\mu}_{\mathrm{cdt}} + u_{\mathrm{cdt}} - \overline{\lambda}_{\mathrm{cdt}} = 0; \quad \forall \mathrm{d}\in\mathcal{D}, t=T \tag{3.100}$$

$$\eta_{\mathrm{dt}} - \overline{\kappa}_{g\mathrm{dt}} + \underline{\kappa}_{g\mathrm{dt}} - C^{gen}_{g\mathrm{dt}} = 0; \forall c\in\mathcal{C}, \mathrm{d}\in\mathcal{D}, t\in\mathcal{T} \tag{3.101}$$

$$\eta_{\mathrm{dt}} - \overline{\delta}_{\mathrm{dt}} + \underline{\delta}_{\mathrm{dt}} - \lambda^{\mathrm{bid}}_{\mathrm{dt}} = 0; \forall c\in\mathcal{C}, \mathrm{d}\in\mathcal{D}, t\in\mathcal{T} \tag{3.102}$$

$$0 \le \underline{\mu}_{\mathrm{cdt}} \ \perp\ 0 \le x_{\mathrm{cdt}} - \underline{P}_{\mathrm{cdt}}; \forall c\in\mathcal{C}, \mathrm{d}\in\mathcal{D}, t\in\mathcal{T} \tag{3.103}$$

$$0 \le \overline{\mu}_{\mathrm{cdt}} \ \perp\ 0 \le \overline{P}_{\mathrm{cdt}} - \Delta^{DR}_{\mathrm{cdt}} - x_{\mathrm{cdt}}; \forall c\in\mathcal{C}, \mathrm{d}\in\mathcal{D}, t\in\mathcal{T} \tag{3.104}$$

$$0 \le \phi^{up}_{\mathrm{cdt}} \ \perp\ 0 \le r^{up}_{\mathrm{cdt}} - x_{\mathrm{cdt}} + x_{\mathrm{cd}t-1}; \forall c\in\mathcal{C}, \mathrm{d}\in\mathcal{D}, t\in\mathcal{T}\setminus\{1\} \tag{3.105}$$

$$0 \le \phi^{\mathrm{d}own}_{\mathrm{cdt}} \ \perp\ 0 \le r^{\mathrm{d}own}_{\mathrm{cdt}} - x_{\mathrm{cd}t-1} + x_{\mathrm{cdt}}; \forall c\in\mathcal{C}, \mathrm{d}\in\mathcal{D}, t\in\mathcal{T}\setminus\{1\} \tag{3.106}$$

$$0 \le \underline{\kappa}_{g\mathrm{dt}} \ \perp\ 0 \le P^{\mathrm{bid},won}_{g\mathrm{dt}}; \forall g\in\mathcal{G}, \mathrm{d}\in\mathcal{D}, t\in\mathcal{T} \tag{3.107}$$

$$0 \le \overline{\kappa}_{g\mathrm{dt}} \ \perp\ 0 \le \overline{P}^{\mathrm{bid}}_{g\mathrm{dt}} - P^{\mathrm{bid},won}_{g\mathrm{dt}}; \forall g\in\mathcal{G}, \mathrm{d}\in\mathcal{D}, t\in\mathcal{T} \tag{3.108}$$

$$0 \le \underline{\delta}_{\mathrm{dt}} \ \perp\ 0 \le \Delta^{\mathrm{bid},won}_{\mathrm{dt}}; \forall \mathrm{d}\in\mathcal{D}, t\in\mathcal{T} \tag{3.109}$$

$$0 \leq \overline{\delta}_{\mathrm{dt}} \ \perp \ 0 \leq \Delta^{\mathrm{bid}}_{\mathrm{dt}} - \Delta^{\mathrm{bid},won}_{\mathrm{dt}}; \forall \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.110}$$

$$\phi^{\mathrm{bid}}_{\mathrm{cdt}}, \phi^{DR}_{\mathrm{cdt}} \in \{0,1\}; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T}, \tag{3.111}$$

where sets $\Omega^1_{\mathrm{dt}} = \{\eta_{\mathrm{dt}}, \overline{\delta}_{\mathrm{dt}}, \underline{\delta}_{\mathrm{dt}}\}$, $\Omega^2_{g\mathrm{dt}} = \{\overline{\kappa}_{g\mathrm{dt}}, \underline{\kappa}_{g\mathrm{dt}}\}$ correspond to the dual variables of the lower level of the reserve market (model (3.27)-(3.30)) and set $\Omega^3_{\mathrm{cdt}} = \{\overline{\mu}_{\mathrm{cdt}}, \underline{\mu}_{\mathrm{cdt}} \phi^{up}_{\mathrm{cdt}} \phi^{down}_{\mathrm{cdt}}\}$, corresponds to the dual variables of loads' optimization problem (model (3.22)-(3.26)).

To reduce the computational burden of the model, complementarity constraints (3.103)-(3.110) can be relaxed by utilizing the Fortuny-Amat transformation [63], obtaining the following mathematical expression:

$$0 \leq \underline{\mu}_{\mathrm{cdt}} \leq M a^{\underline{\mu}}_{\mathrm{cdt}}; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.112}$$

$$x_{\mathrm{cdt}} - \underline{P}_{\mathrm{cdt}} \leq (1-M) a^{\underline{\mu}}_{\mathrm{cdt}}; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.113}$$

$$0 \leq \overline{\mu}_{\mathrm{cdt}} \leq M a^{\overline{\mu}}_{\mathrm{cdt}}; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.114}$$

$$\overline{P}_{\mathrm{cdt}} - \Delta^{DR}_{\mathrm{cdt}} - x_{\mathrm{cdt}} \leq (1-M) a^{\overline{\mu}}_{\mathrm{cdt}}; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.115}$$

$$0 \leq \phi^{up}_{\mathrm{cdt}} \leq M a^{\phi^{up}}_{\mathrm{cdt}}; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.116}$$

$$r^{up}_{\mathrm{cdt}} - x_{\mathrm{cdt}} + x_{\mathrm{cdt}-1} \leq (1-M) a^{\phi^{up}}_{\mathrm{cdt}}; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.117}$$

$$0 \leq \phi^{\mathrm{down}}_{\mathrm{cdt}} \leq M a^{\phi^{\mathrm{down}}}_{\mathrm{cdt}}; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.118}$$

$$r^{\mathrm{down}}_{\mathrm{cdt}} - x_{\mathrm{cdt}-1} + x_{\mathrm{cdt}} \leq (1-M) a^{\phi^{\mathrm{down}}}_{\mathrm{cdt}}; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.119}$$

$$0 \leq \underline{\kappa}_{g\mathrm{dt}} \leq M a^{\underline{\kappa}}_{g\mathrm{dt}}; \forall g \in \mathcal{G}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.120}$$

$$P^{\mathrm{bid},won}_{g\mathrm{dt}} \leq (1-M) a^{\underline{\kappa}}_{g\mathrm{dt}}; \forall g \in \mathcal{G}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.121}$$

$$0 \leq \overline{\kappa}_{g\mathrm{dt}} \leq M a^{\overline{\kappa}}_{g\mathrm{dt}}; \forall g \in \mathcal{G}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.122}$$

$$\overline{P}^{\mathrm{bid}}_{g\mathrm{dt}} - P^{\mathrm{bid},won}_{g\mathrm{dt}} \leq (1-M) a^{\overline{\kappa}}_{g\mathrm{dt}}; \forall g \in \mathcal{G}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.123}$$

$$0 \leq \underline{\delta}_{\mathrm{dt}} \leq M a^{\underline{\delta}}_{\mathrm{dt}}; \forall \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.124}$$

$$\Delta^{\mathrm{bid},won}_{\mathrm{dt}} \leq (1-M) a^{\underline{\delta}}_{\mathrm{dt}}; \forall \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.125}$$

$$0 \leq \overline{\delta}_{\mathrm{dt}} \leq M a^{\overline{\delta}}_{\mathrm{dt}}; \forall \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.126}$$

$$\Delta^{\mathrm{bid}}_{\mathrm{dt}} - \Delta^{\mathrm{bid},won}_{\mathrm{dt}} \leq (1-M) a^{\overline{\delta}}_{\mathrm{dt}}; \forall \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{3.127}$$

$$a^{\underline{\mu}}_{\mathrm{cdt}}, a^{\overline{\mu}}_{\mathrm{cdt}}, a^{\phi^{up}}_{\mathrm{cdt}}, a^{\phi^{\mathrm{down}}}_{\mathrm{cdt}}, a^{\underline{\kappa}}_{g\mathrm{dt}}, a^{\overline{\kappa}}_{g\mathrm{dt}}, a^{\underline{\delta}}_{\mathrm{dt}}, a^{\overline{\delta}}_{\mathrm{dt}} \in \{0,1\}; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T}, \tag{3.128}$$

Finally, the bilinear term $\lambda^{bid}_{\mathrm{dt}}\Delta^{\mathrm{bid},won}_{\mathrm{dt}}$ in the objective function of the upper level (3.92) can be relaxed using McCormick envelopes [64]:

$$
\begin{aligned}
&\underset{\substack{\Delta^{DR}_{\mathrm{cd}t},\lambda^{\mathrm{bid}}_{\mathrm{d}t},\Delta^{\mathrm{bid},won}_{\mathrm{d}t},\\ x_{\mathrm{cd}t},\phi^{DR}_{\mathrm{cd}t},\phi^{\mathrm{bid}}_{\mathrm{cd}t},\Delta^{\mathrm{bid}}_{\mathrm{d}t},\\ \Omega^{1}_{\mathrm{d}t},\Omega^{2}_{g\mathrm{d}t},\Omega^{3}_{\mathrm{cd}t},\\ a^{\underline{\mu}}_{\mathrm{cd}t},a^{\overline{\mu}}_{\mathrm{cd}t},a^{\phi^{up}}_{\mathrm{cd}t},a^{\phi^{\mathrm{down}}}_{\mathrm{cd}t},\\ a^{\underline{\kappa}}_{g\mathrm{d}t},a^{\overline{\kappa}}_{g\mathrm{d}t},a^{\underline{\delta}}_{\mathrm{d}t},a^{\overline{\delta}}_{\mathrm{d}t}}}{\text{Maximize}} \quad \sum_{c\in\mathcal{C}}\sum_{\mathrm{d}\in\mathcal{D}}\sum_{t\in\mathcal{T}}\Delta^{DR}_{\mathrm{cd}t}\left(\lambda^{E}_{\mathrm{d}t}-\overline{\lambda}_{\mathrm{cd}t}\right)+\sum_{\mathrm{d}\in\mathcal{D}}\sum_{t\in\mathcal{T}}O_{\mathrm{d}t} && (3.129)\\
&\text{subject to:}\\
&\text{Constraints } (3.93)-(3.102) \text{ and } (3.112)-(3.128) && (3.130)\\
&\underline{\lambda}^{\mathrm{bid}}_{\mathrm{d}t}\Delta^{\mathrm{bid},won}_{\mathrm{d}t}+\lambda^{\mathrm{bid}}_{\mathrm{d}t}\underline{\Delta}^{\mathrm{bid},won}_{\mathrm{d}t}\underline{\lambda}^{bid}_{\mathrm{d}t}\underline{\Delta}^{\mathrm{bid},won}_{\mathrm{d}t}\leq O_{\mathrm{d}t};\\
&\qquad\qquad\qquad\qquad \forall \mathrm{d}\in\mathcal{D},t\in\mathcal{T} && (3.131)\\
&\overline{\lambda}^{bid}_{\mathrm{d}t}\Delta^{\mathrm{bid},won}_{\mathrm{d}t}+\lambda^{bid}_{\mathrm{d}t}\overline{\Delta}^{\mathrm{bid},won}_{\mathrm{d}t}-\overline{\lambda}^{bid}_{\mathrm{d}t}\overline{\Delta}^{\mathrm{bid},won}_{\mathrm{d}t}\leq O_{\mathrm{d}t};\\
&\qquad\qquad\qquad\qquad \forall \mathrm{d}\in\mathcal{D},t\in\mathcal{T} && (3.132)\\
&\overline{\lambda}^{bid}_{\mathrm{d}t}\Delta^{\mathrm{bid},won}_{\mathrm{d}t}+\lambda^{\mathrm{bid}}_{\mathrm{d}t}\underline{\Delta}^{\mathrm{bid},won}_{\mathrm{d}t}-\underline{\lambda}^{bid}_{\mathrm{d}t}\overline{\Delta}^{\mathrm{bid},won}_{\mathrm{d}t}\geq O_{\mathrm{d}t};\\
&\qquad\qquad\qquad\qquad \forall \mathrm{d}\in\mathcal{D},t\in\mathcal{T} && (3.133)\\
&\lambda^{\mathrm{bid}}_{\mathrm{d}t}\overline{\Delta}^{\mathrm{bid},won}+\underline{\lambda}^{\mathrm{bid}}_{\mathrm{d}t}\Delta^{\mathrm{bid},won}_{\mathrm{d}t}-\underline{\lambda}^{\mathrm{bid}}_{\mathrm{d}t}\overline{\Delta}^{\mathrm{bid},won}_{\mathrm{d}t}\geq O_{\mathrm{d}t};\\
&\qquad\qquad\qquad\qquad \forall \mathrm{d}\in\mathcal{D},t\in\mathcal{T} && (3.134)\\
&\underline{\lambda}^{bid}_{\mathrm{d}t}\leq\lambda^{\mathrm{bid}}_{\mathrm{d}t}\leq\overline{\lambda}^{\mathrm{bid}}_{\mathrm{d}t};\forall \mathrm{d}\in\mathcal{D},\forall t\in\mathcal{T} && (3.135)\\
&\underline{\Delta}^{\mathrm{bid},won}_{\mathrm{d}t}\leq\Delta^{\mathrm{bid},won}_{\mathrm{d}t}\leq\overline{\Delta}^{\mathrm{bid},won}_{\mathrm{d}t};\forall \mathrm{d}\in\mathcal{D},t\in\mathcal{T}, && (3.136)
\end{aligned}
$$

where parameters $\underline{\Delta}^{\mathrm{bid},won}_{\mathrm{d}t}$, $\overline{\Delta}^{\mathrm{bid},won}_{\mathrm{d}t}$, $\underline{\lambda}^{\mathrm{bid}}_{\mathrm{d}t}$ and $\overline{\lambda}^{\mathrm{bid}}_{\mathrm{d}t}$ represent upper and lower bounds for variables $\lambda^{\mathrm{bid}}_{\mathrm{d}t}$ and $\Delta^{\mathrm{bid},won}_{\mathrm{d}t}$ respectively, and $O_{\mathrm{d}t}$ consists in the ancillary variable of the bilinear term.

Chapter 4

Chilean Electricity Market Case Study

4.1 Input Data

In this section, we present the main inputs used to validate our proposed framework and to determine the economic benefits of a multi-service approach. For this work, we use real-world data corresponding to the Chilean energy and reserve markets [60] and the demand profiles of large-sized loads managed by an electricity retailer in Santiago, Chile [65].

4.1.1 Energy Market

We use time series of day-ahead energy prices with hourly resolution from the Chilean electricity market, ranging from October 1st, 2019 to September 30th, 2020 [60]. Price profiles from 2019 are used as a training set to forecast loads' energy output during the first months of 2020, while 2020 price profiles are used as the core data-set throughout all case studies. Energy price boxplots for each hour of the day are shown in Fig. 4.1.

4.1.2 Reserve Market

We use the data corresponding to the Chilean upward tertiary reserve market through January 1st to September 30th, 2020 [60]. Tertiary reserves are mainly used by the Chilean ISO to correct forecasting errors and/or release secondary reserves, with a required duration once activated of 1 hour. We formulate this market using a pay-as-bid auction scheme, where participants submit one bid per hour of the day and are remunerated based on their bidding price. Market bids are composed of the amount offered as upward tertiary reserves and a specific price. Once all bids have been submitted, the market is cleared by the ISO in an hourly fashion to meet reserve requirements for the next day. Table 4.1 shows the tertiary reserve requirements (R_t) for 2020, where 5 time windows are set by the ISO and the requirements for each window remain fixed throughout the year. To reduce the computational burden of the model, all generator-based bids are clustered into 12 representative agents since no more than 12 generators participated simultaneously during 2020 [60]. Finally, we consider an activation rate of $\rho = 5\%$ for placed bids to represent reserve activation during real-time operation (see equation (3.21)).

4.1.3 Electricity Retailer and Load Data

Three time series with an hourly resolution corresponding to large-sized loads are provided by an electricity retailer located in the main load center at Santiago, Chile [65]. In a similar fashion to the price series of the energy market, load data ranges from October 1st, 2019 to September 30th, 2020, and its energy profiles are shown in Fig. 4.2. For each load, the contract price is \$63.44, contracted DR amount $\overline{\Delta}_c$ is set at 0.75 [MW], and maximum calls per day is set at 1. Finally, we assume that the flexible load contract can be utilized a maximum of 52 times per year per load and is evenly distributed throughout all quarters of 2020 ($\overline{\phi}_c$ is set at 13 per quarter).

Table 4.1: Tertiary Reserve daily requirements

Time Window	00:00 - 06:00	06:00 - 10:00	10:00 - 17:00	18:00 - 21:00	21:00 - 00:00
Reserve Requirements [MW]	226	263	208	334	226

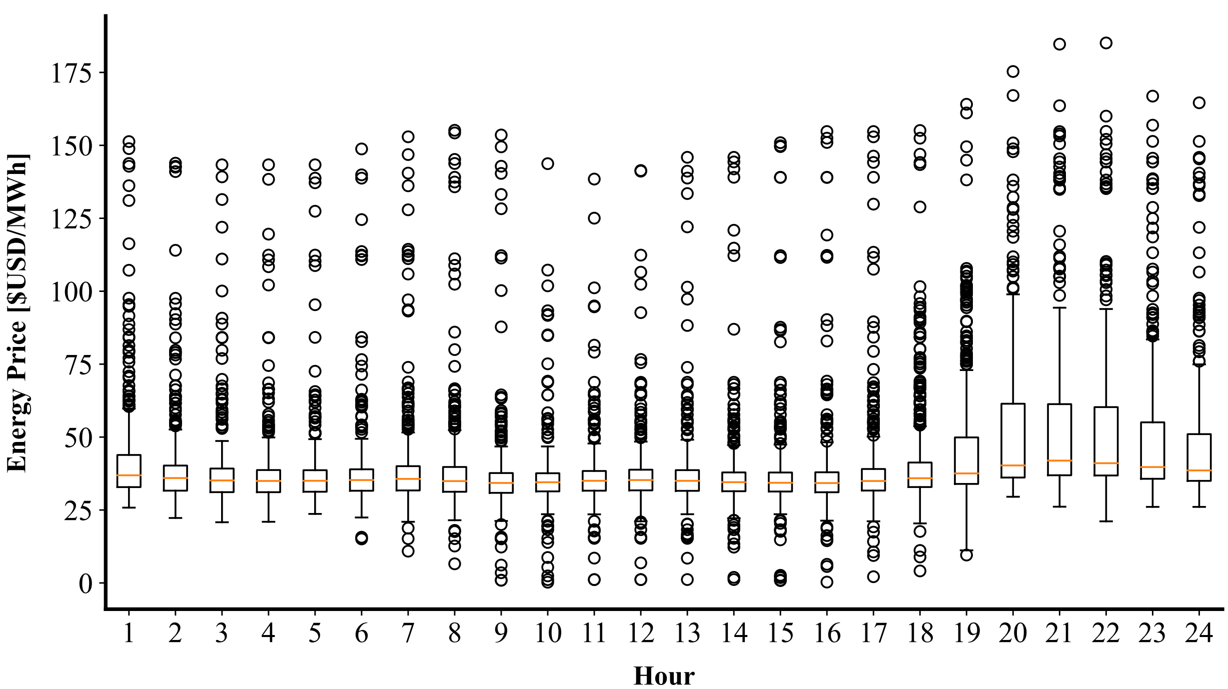

Figure 4.1: Hourly energy price boxplots.

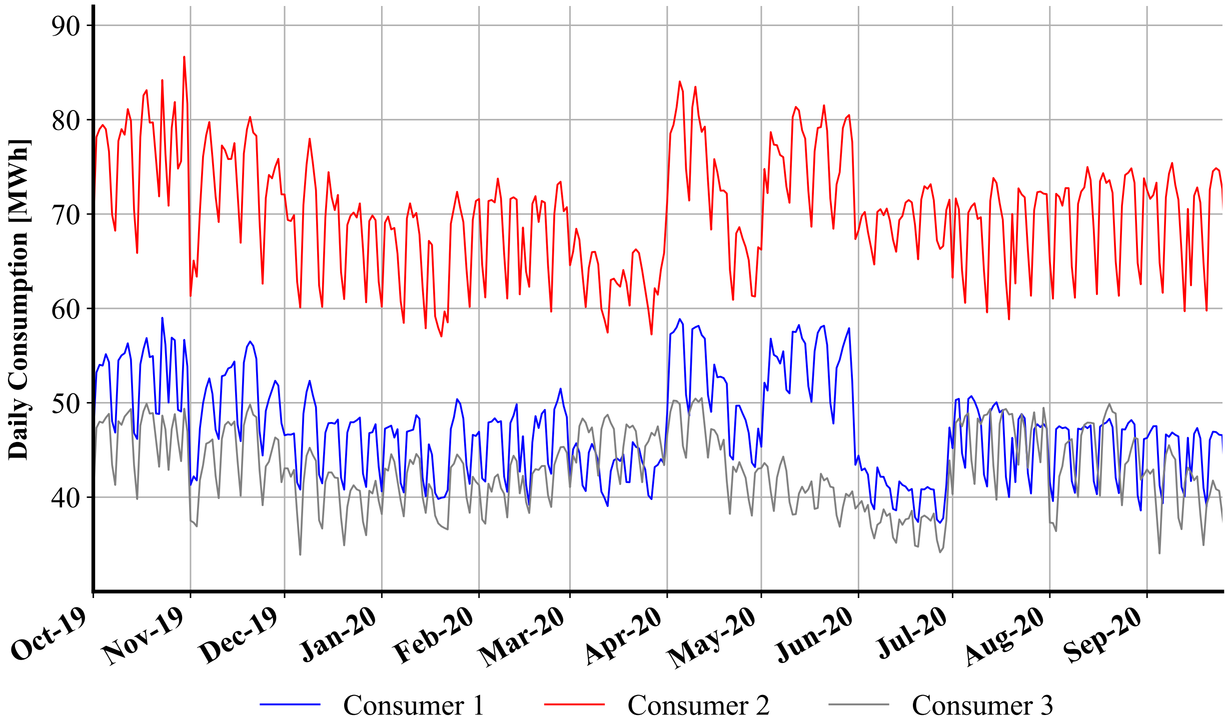

Figure 4.2: Daily consumption profiles for each large-sized load. Data ranges from October 1st, 2019 to September 30th, 2020.

4.1.4 Case Studies

Using the aforementioned input data, we carry out several case studies to validate the proposed framework and to determine the value of utilizing flexible load contracts by offering DR-based services in energy and reserve markets:

- As a starting point, we validate the first stage of the model by forecasting the energy consumption of one large-sized load and comparing the results with other commonly used forecasting methods.
- Once the first stage is validated, we proceed to validate the second stage by running our model from January 1st to September 30th, 2020 and demonstrating: (i) the economic benefits perceived by the retailer, (ii) the robustness of our two-stage methodology upon varying activation rates ρ and flexible load contract volumes $\overline{\Delta}_c$, (iii) the value of demand-side participation in the tertiary reserve markets through the economic benefits perceived by the ISO, and (iv) the ability of flexible loads to provide multiple DR-based services to the system.

The model is executed using Julia 1.5 and Gurobi [66] on a machine with 16GB of RAM and 8-core processors.

4.2 Results and Discussion

4.2.1 Validation of Stage I

In this section, we seek to validate the first stage of the model, which consists of estimating the parameters of loads' decision-making process and predicting its energy output. In this context, we select one large-sized load and validate our model by forecasting its energy consumption from January 1st to September 30th, 2020. The forecasting method is done in a rolling-horizon manner, where energy output is predicted for the following 7 days using the last 90 days of consumption as a training set.

Regarding the parameters of the proposed model, we consider 14 regressors in our feature vector $z_{c\mathrm{d}t}$, namely the energy price and energy consumption of the last 6 days (i.e. $\lambda^{DA}_{c(\mathrm{d-i})t}$, $x_{c(\mathrm{d-i})t}$ $\forall \mathrm{i} \in \{1...6\}$), contract price $\overline{\lambda}_{c\mathrm{d}t}$, and a binary variable that identifies if a specific day corresponds to a weekday or weekend. Finally, penalty parameter $L_{c\mathrm{di}}$ and Kernel scaling parameter γ are tuned for each 7-day forecast horizon using a grid search technique based on [56], where we solve model (3.72)-(3.83) for the 90-day training set using different combinations of $L_{\mathrm{i}st}$ and γ and utilize the pair that minimizes out-of-sample error for the following 7-day horizon.

For validation purposes, we analyze the forecasting capabilities of our proposed model (Kernel-IO) and compare the results against three commonly used forecasting models, namely, Auto-regressive model with exogenous variables [67] (ARX), Support Vector Machine [68] (SVM) and IO without exogenous variables [52] (Regular-IO). For the ARX and SVM models, we deploy the same regressors used in the Kernel-IO model and SVM parameters are tuned using the aforementioned grid search technique.

Fig. 4.3 shows the forecasting results of all models for a sample of 7 consecutive days, ranging from the 30th of April to the 6th of May, 2020. Firstly, note that the Regular-IO model under-performs in terms of its fitting capabilities compared to other forecasting models. This is due to the lack of a feature vector, which forces the model to make predictions based on the assumption that estimated parameters derived from the training set $\mathcal{S}$ will remain fixed throughout future operating scenarios (i.e. $u_{cst} = u_{c\mathrm{d}t}$, $\forall s \in \mathcal{S}, \mathrm{d} \in \mathcal{D}$), and therefore solely relying on the tuning of penalty parameter $L_{\mathrm{i}st}$ to adjust for varying consumption levels. On the other hand, the ARX and SVM models show better fitting capabilities but tend to underestimate peak consumption, especially during days of high demand. Results also demonstrate that the Kernel-IO model shows the best fitting capabilities and can adjust its consumption through different days with suitable out-of-sample errors. Finally, Table 4.2 shows the average error metrics throughout 2020, namely, mean absolute percentage error (MAPE), root mean square error (RMSE), and mean absolute error (MAE). These results further demonstrate that the Kernel-IO approach outperforms the other three forecasting methods while also having the advantage of reconstructing the optimization problem solved by loads and using it to represent load behavior when optimizing retailers' operations.

Table 4.2: Average error metrics for each model throughout 2020.

Model	MAPE	MAE	RMSE
Kernel-IO	8.96	17.54	5.59
Regular-IO	10.99	21.48	9.56
SVM	10.28	20.62	5.65
ARX	11.91	23.40	13.23

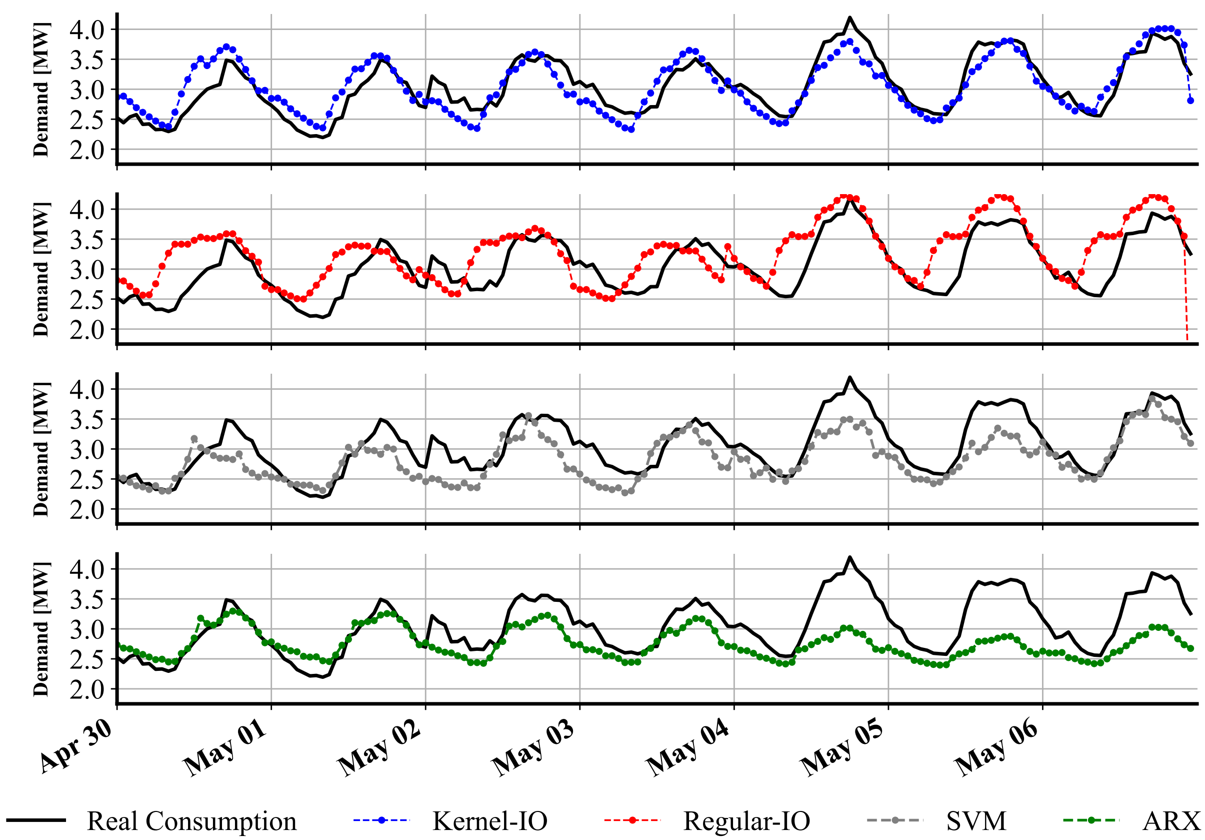

Figure 4.3: Forecasting results during Apr. 30 - May. 06, 2020. Results for each model are indicated by a distinctive color while the black line corresponds to actual consumption.

4.2.2 Validation and Application of Stage II

In this section, we proceed to run our two-stage model throughout the first three quarters of 2020 (from January 1st to September 30th), and analyze the value of a multi-service approach by studying the economic benefits perceived by the retailer compared to solely participating in the energy market, the robustness our methodology through sensitivity analyses on the activation rate ρ and contract volume $\overline{\Delta}_c$, and the economic benefits to the ISO upon enabling demand-side resources to participate in the reserve market. In addition, we study a representative week to validate the ability of flexible loads to provide multiple DR-based services to the system.

In regards to the parameters used in this section, for the first stage we maintain the same 14 regressors used in Section 4.2.1, which correspond to the energy price and consumption levels of the previous 6 days, contract price $\overline{\lambda}_{\mathrm{cd}t}$, and the week-day / weekend binary variable. In addition, penalty parameter L_{ist} and Kernel coefficient γ are tuned using the same grid search technique specified in Section 4.2.1. Once the first stage is solved and load parameters are estimated for future operating scenarios, the results are inputted in model (3.22)-(3.26) to characterize load behavior in the second stage. The following sections provide the results and discussion for each case.

Retailer benefits

The first step used in validating the second stage corresponds to running the model throughout the first three quarters of 2020 and comparing the results of each quarter against a base case where the retailer can only participate in the energy market. In this base case, the retailer still has the same flexible load volume $\overline{\Delta}_c$ and contract price $\overline{\lambda}_{\mathrm{cd}t}$ for its load portfolio. However, it cannot place any bids in the reserve market and can only utilize its flexible load contracts to reduce procurement costs in the energy market. The parameters used for both cases correspond to the ones detailed in Section 4.1.

Table 4.3 shows the economic benefits perceived by the retailer when utilizing its flexible load contracts only in the energy market (base case), or in both energy and tertiary reserve markets (proposed model). In this case, the economic results are divided into two main groups. The first component corresponds to the savings derived from the energy market, which is calculated based on the price difference between the day ahead price of the energy market, $\lambda_{\mathrm{d}t}^{E}$ and flexible loads' contract price, $\overline{\lambda}_{\mathrm{cd}t}$, multiplied by the reduction in electricity consumption $\Delta_{\mathrm{cd}t}^{DR}$ for each load. On the other hand, the second revenue stream is derived from the pay-as-bid remuneration mechanism of the reserve market, which (in the case of the retailer) corresponds to the bidding price, $\lambda_{\mathrm{d}t}^{\mathrm{bid}}$ multiplied by the amount of reserves won, $\Delta_{\mathrm{d}t}^{\mathrm{bid},won}$. Note that the pay-as-bid remuneration mechanism of the reserve market awards market participants based on reserve availability, without necessarily deploying the reserves during real-time operation.

Regarding the base case, Table 4.3 shows that the retailer provides 78 [MWh] of DR in the energy market, generating savings of $4,997 throughout the time horizon. In this case, since the retailer can only utilize its flexible load contracts in the energy market, there is no remuneration derived from the reserve market.

On the other hand, Table 4.3 shows that the the retailer in the proposed model provides 52.35 [MWh] of DR-based services in the energy market throughout all quarters, generating total savings of $4,817. In addition, by leveraging its ability to participate in the reserve market, the retailer also utilizes its flexible load contracts to provide (on average) 2.25 [MW] of reserves during 241 hours, totaling 542.25 [MWh] throughout the time horizon and receiving a total revenue of $16,488.

These results demonstrate that the retailer can achieve significant economic benefits when utilizing its flexible load contracts in energy and reserve markets. In this regard, the proposed model generates achieves $19,957 in total benefits compared to $4,997 of the base case. This difference between the economic performance of both models is due to the payment mechanism of the tertiary reserve market, where winning bids receive payment for being on-hold (capacity payment) and without necessarily having to deploy their reserves during real-time operation. Therefore, high capacity payments coupled with the low activation rates of the tertiary reserve market allow the retailer to achieve higher economic benefits compared to solely participating in the energy market.

Table 4.3: Revenue per quarter of 2020 (Q) for each model

Item		**Q1**	**Q2**	**Q3**
Proposed Model	DR in energy market [MWh]	17.27	18.01	17.07
	DR for reserve provision [MWh]	180.02	180.01	183.00
	Savings energy market [$]	1,718	917	834
	Revenue reserve market [$]	3,259	7.103	6,126
	Total economic benefits [$]	4,977	8,020	6,960
Base Case	DR in energy market [MWh]	24.37	27.29	26.95
	Savings energy market [$]	2,310	1,573	1,114

Sensitivity analyses

Results shown in the previous section demonstrate that the retailer can achieve significant economic benefits when utilizing its flexible load contracts in energy and reserve markets. However, the remuneration obtained from participating in the reserve market is highly dependent on the activation rate ρ (see equation (3.36)), due to the fact that the retailer has the capability to bid during multiple instances without necessarily having to utilize its flexible load contracts as reserves during real-time operation. For instance, an activation rate of $\rho = 5\%$ means that the retailer has a 5% probability that awarded reserves are needed during real-time operation, forcing the retailer to utilize its flexible load contracts to provide reserves to the system. As a consequence, if the the expected activation rate of reserves is $\rho = 5\%$, the retailer has the option to participate up to 20 times in the reserve market before having to utilize its flexible load contracts for reserve provision.

Therefore, it is necessary to carry out a sensitivity analysis on the activation rate parameter ρ in order to determine how the retailer decides to utilize its flexible load contracts upon different levels of activation rates. In this regard, we proceed to run the model using the same conditions of the previous case, but with activation rates ranging from the original value of $\rho = 5\%$ to a highly conservative value of $\rho = 40\%$.

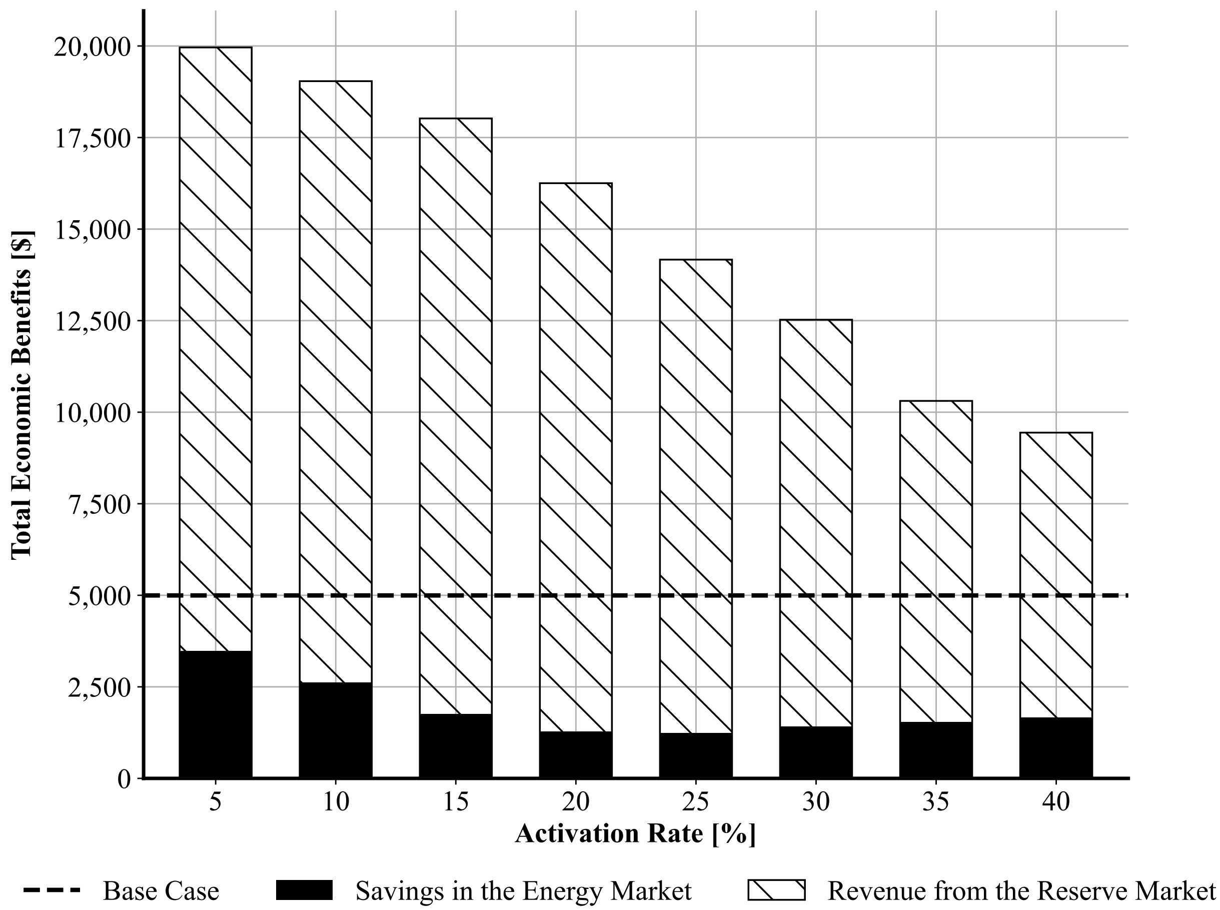

Figure 4.4: Economic benefits perceived by the retailer upon varying activation rates of the reserve market, ranging from 5% to 40%. The dashed black line corresponds to the base case where only participation in the energy market is allowed.

Fig. 4.4 shows the total economic benefits (from January 1st to September 30th, 2020) perceived by the retailer for several activation rates ρ. The results show that the retailer still maintains a high utilization rate of its flexible load contracts for reserve provision, where more than 85% of the total economic benefits from its flexible load portfolio is derived from revenues associated to the reserve market. In addition, by comparing the results of the proposed model against the results obtained from the base case (dashed black line in Fig. 4.4), the multi-service approach economically outperforms the base case even in high activation rate scenarios such as $\rho = 40\%$, where the proposed approach generates $9,442 in total benefits compared to $4,997 of the base case.

In a similar fashion to the results shown in the previous section, the economic outperformance of the proposed model is mostly due to the high capacity payments of the reserve market. Even in high activation rate scenarios such as $\rho = 40\%$, the model is still capable of participating up to two times in the reserve market before needing to utilize its flexible load resources for reserve provision during real-time operation. This allows the retailer to perceive higher economic benefits if compared to solely participating in the reserve market, since in this market the retailer is forced to utilize its flexible load contracts for each operation.

Hence, this sensitivity on the activation rate of reserves demonstrates that, despite the fact that the retailer faces reduced revenues upon high reserve activation rates, it still economically outperforms the base case where the retailer cannot participate in the reserve market.

Table 4.4: Overall results for several flexible load contract volumes $\overline{\Delta}_c$

Item	**Total contract volume [MW]** $\overline{\Delta}_c = \mathbf{10}$	$\overline{\Delta}_c = \mathbf{50}$	$\overline{\Delta}_c = \mathbf{100}$	$\overline{\Delta}_c = \mathbf{150}$
DR in energy market [MWh]	199.79	993.68	1,987.56	2,981
DR for reserve provision [MWh]	2,428.02	12,140.01	24,273.79	36,405.76
Percentage of total DR in energy market [%]	7.61	7.57	7.57	7.57
Percentage of total DR for reserve provision [%]	92.39	92.43	92.43	92.43
Savings energy market [$]	9,287	46,796	94,023	141,260
Revenue reserve market [$]	55,304	271,200	531,040	777,450
Total economic benefits [$]	64,591	317,996	625,063	918,710
Savings energy market per MW of flexible load contract $\overline{\Delta}_c$ [$/MW]	929	936	940	942
Revenue reserve market per MW of flexible load contract $\overline{\Delta}_c$ [$/MW]	5,530	5,424	5,310	5,183
Total economic benefits per MW of flexible load contract $\overline{\Delta}_c$ [$/MW]	6,459	6,310	6,250	6,125

In addition to carrying out a sensitivity analysis on reserve activation rate ρ, a second sensitivity on flexible load contract volume $\overline{\Delta}_c$ is carried out in order to further analyze how the retailer utilizes its contracted flexibility and at the same time determine the most economically efficient size of its flexible load portfolio.

In this vein, the second sensitivity analysis carried out in this work consists on maintaining the base parameters utilized in previous sections, but with a variation of contract volumes ranging from $\overline{\Delta}_c = 10$ [MW] to $\overline{\Delta}_c = 150$ [MW]. Regarding market parameters, the activation rate of the reserve market is maintained at the reference case of $\rho = 5\%$.

Table 4.4 shows the overall results when increasing contract volumes $\overline{\Delta}_c$ from 10 [MW] to 150 [MW]. As expected, the bigger the volume of flexible load contracts, the higher the economic benefits derived from each market, which is caused by the increased volume of DR-based services offered in each market. In addition, Table 4.4 shows that despite the increased amount of contract volumes, the total utilization percentage in each market remains relatively the same. In a similar fashion to previous analysis, the utilization rate between markets is mostly derived from the high capacity payments of the reserve market, where the retailer has the capability to participate (and be remunerated) multiple times without necessarily utilizing its flexible load contracts for reserve provision.

However, one salient feature of the results shown in Table 4.4 correspond to the economic benefits perceived by the retailer per MW of flexible load available. In this set of results, Table 4.4 shows that savings (per MW of contracted volume $\overline{\Delta_c}$) in the energy market tend to increase when a larger volume of flexible contracts is available, while revenues derived from the reserve market present a downward trend. Additionally, the total economic benefits per MW of contracted volume tend to decrease above the threshold of 50 [MW].

These trends shown in Table 4.4 are mostly due to the reduction of the bidding price when offering higher volumes in the tertiary reserve market. As contracted volume $\overline{\Delta_c}$ increases, the retailer tends to place larger quantities when participating in the tertiary reserve market (maintaining a 92.4% of its total DR provision between both markets). However, since larger volumes are offered in the reserve market and market participants can only place one bid per hour of the day, the retailer has to lower its bidding price to be awarded the total volume that was bid in the market, therefore generating an overall reduction in the total revenue per MW of contracted flexible load.

Hence, this demonstrates that from the retailers' perspective, the results obtained by the proposed model are reasonably robust against changes in total volume, where the economic benefits remain despite the increase in volume of the contracted flexible load portfolio. However, this sensitivity also demonstrates that the retailer can potentially experience diminishing returns per MW of contracted flexible loads. This is mostly due to the fact that as contracted volume increases, the retailer is forced to lower its price bits in the reserve market to be awarded its full offering capacity. Additionally, the results shown in Table 4.4 also demonstrate that the volume of retailers' flexible load contracts is upper-bounded, due to the presence of diminishing returns beyond a specific threshold.

ISO benefits

As mentioned on the previous analysis, sensitivities on activation rates ρ and contracted flexible load volume $\overline{\Delta_c}$ show that first, the model outperforms the base case even with high activation rates, and second, the model tends to participate more in the reserve market as the contracted volume increases. However, since the retailer has the option to strategically bid in the reserve market (and therefore have a direct influence on market prices), it is relevant to study how increasing contracted volumes for the retailer may influence the reserve costs perceived by the ISO.

In this context, we proceed to run the two-stage model throughout the first three quarters of 2020 (from January 1st to September 30st) with varying contract volumes $\overline{\Delta_c}$, while maintaining the activation rate of the reserve market at the reference value of $\rho = 5\%$. This methodology will allow us to study the impact on auction prices and reserve costs when the retailer can displace a bigger portion of the supply curve due to having a larger volume of demand-side resources to offer to the system.

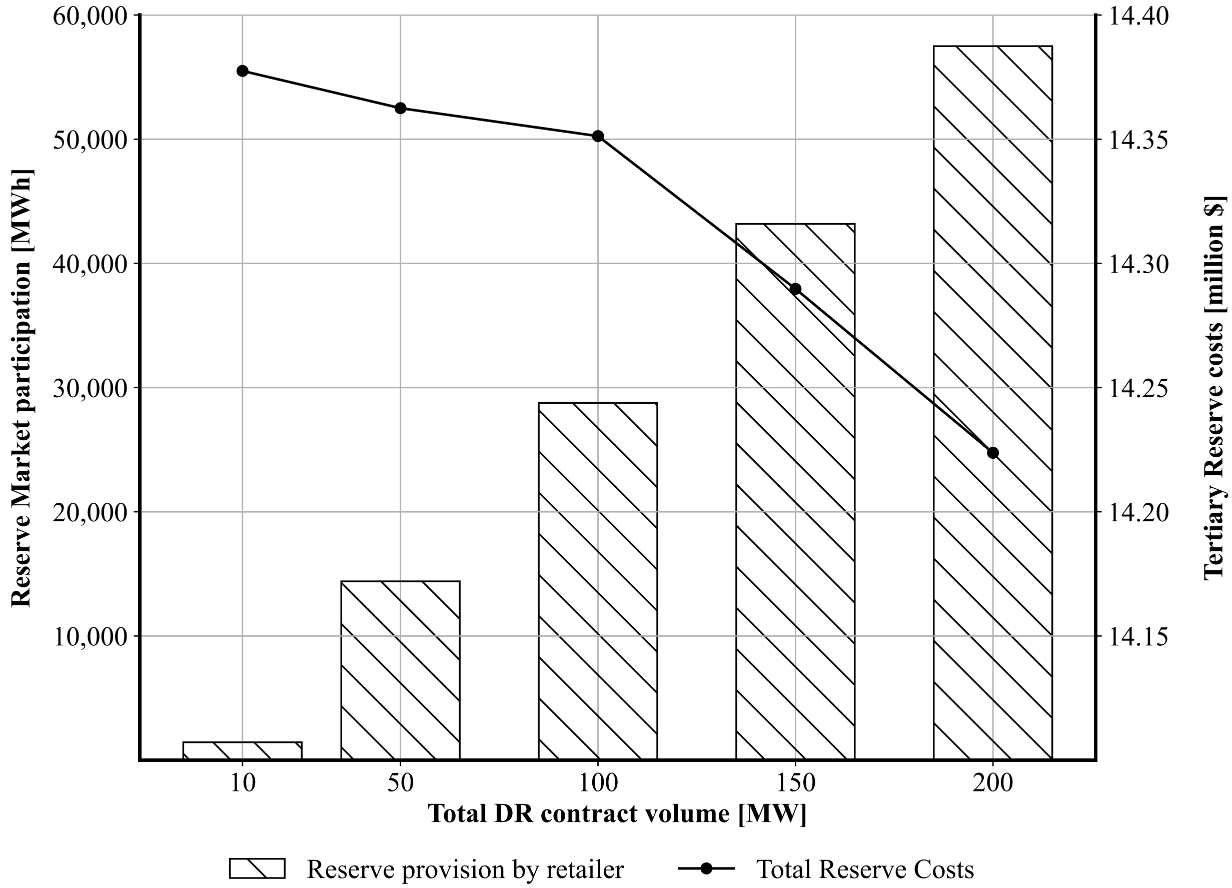

Figure 4.5: Cost reduction upon increasing demand-side participation. The black line corresponds to the reserve costs perceived by the ISO when buying the cheapest bids in the auction-based tertiary reserve market.

Fig. 4.5 demonstrates the cost reduction perceived by the system operator when enabling demand-side participation in the tertiary reserve market, and how varying contract volumes impact total reserve costs to the system. In effect, Fig. 4.5 shows a correlation between flexible contract volume $\overline{\Delta}_c$ and total reserve costs, where an increase in contract volume up to 200 [MW] generates a cost reduction up to \$153,750.

This cost reduction is mostly due to the relationship between the market conditions of the reserve market and the contracted flexible load volume. As analyzed in section 4.2.2, activation rates coupled with high capacity payments of the tertiary reserve market allow the retailer to achieve significant economic benefits compared to the energy market. As contract volume $\overline{\Delta}_c$ increases, the economic benefits of the reserve market drive the retailer to place larger quantities into the reserve market. However, since market participants can only place one price-quantity bid per hour of the day, the retailer is forced to lower its bidding price to be awarded its full offering capacity. Therefore, the reduction in offering price causes the retailer to displace the most expensive generator-based bids and drive down the reserve acquisition costs for the ISO.

This demonstrates that enabling demand-side resources in the reserve market has the capability of reducing reserve costs to the ISO due to the displacement effect of the most expensive generator-based bids. However, as seen in Section 4.2.2, the retailer experiences diminishing returns per MW of contracted volume upon a certain threshold that is below the market size of the reserve market. Hence, the potential reduction of reserve costs is upper-bounded since the contracted amount of flexible loads will not allow for the full displacement of the generator-based supply curve as it is not economically efficient.

Finally, note that Fig. 4.5 shows that enabling demand-side participation only generates a 1.06% reduction of reserve costs. This is due to the limitations imposed by our modeling framework, where the retailer can only place one bid per day, and a limited number of times per year. However, by analyzing reserve costs only considering the hours where there was demand-side participation, the total cost reduction derived from enabling DR corresponds to 8.91% of reserve costs.

Flexible load operation

The last component of the proposed case study consists of analyzing the operation of our proposed two-stage model. To do so, we run our model using the reference parameters illustrated in section 4.1, and proceed to select one representative week that shows the complete operation carried out by the retailer in both energy and reserve markets and how its decision-making process impacts the consumption patterns of flexible loads.

Fig. 4.6 illustrates the operation of a flexible through a representative week, where the retailer leverages loads' flexibility to participate in energy and reserve markets. As seen in Fig. 4.6, throughout the week the retailer utilizes its flexible load contract to participate 3 times in the tertiary reserve market, and the load is capable of providing up to 2.25 [MW] in tertiary reserve capacity (as availability). In addition to its participation in the reserve market, the retailer also utilizes loads' flexibility to reduce consumption levels, totaling 3 [MW] of curtailed energy in the energy market.

This shows that the retailer has the capability to leverage the capacity payment mechanism of the reserve market and be awarded reserve availability rights without necessarily having to utilize its flexible load contracts during real time operation. In addition, this further demonstrates the flexibility of our proposed approach, which allows the retailer to participate in the reserve market throughout the week while also having the option to utilize its flexible contracts to provide DR in the energy market if market conditions are not favorable.

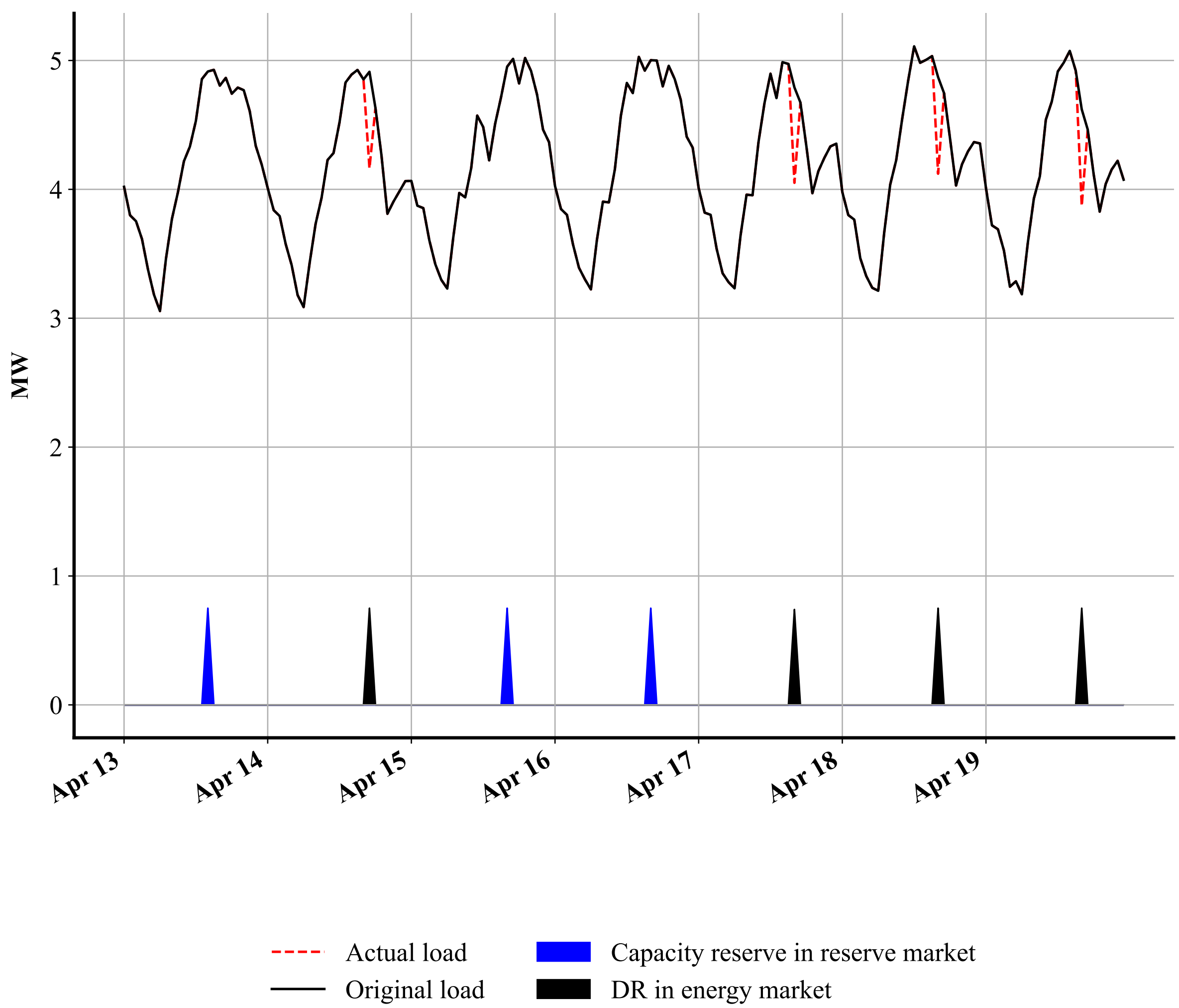

Figure 4.6: Flexible load operation during one representative week (Apr. 13 - Apr. 19), where it provides energy and reserve DR-based services.

Chapter 5

Conclusions and Further Work

5.1 Conclusions

This work proposed a framework to determine the operation of a retailer managing a portfolio of flexible loads that offers DR-based services in day-ahead energy and reserve wholesale markets. To do so, we proposed a two-stage model. The first stage represents consumer behavior through a linear optimization problem, and estimates its parameters for future operating scenarios using IO. The second stage deploys a bilevel scheme to optimize retailers' operations through the profit-maximizing utilization of its flexible load contracts.

Overall, our proposed two-stage model allows the retailer to reduce the electricity consumption of its flexible demand portfolio through DR services in the energy market, as well as aggregating its flexible load contracts to offer DR-based services in an auction-based reserve market. In regards to the latter, the retailer is modeled as a strategic agent that has the capability to alter auction prices through its price-quantity bids.

By testing our two-stage model through several case studies using the Chilean electricity market, we first demonstrated the characterization capabilities of using IO as a tool to model load behavior. In this regard, the model was capable of forecasting the energy consumption of large-sized loads while outperforming other commonly used forecasting methods in terms of out-of-sample error metrics. These results demonstrate that the IO approach can be used as a successful forecasting tool with the additional advantage that recovered parameters can later be used to characterize load behavior when optimizing retailers' operations.

In addition to validating the first stage and demonstrating the forecasting capabilities of using an IO-based approach, we also demonstrated the value of utilizing DR-based services in energy and reserve markets by analyzing the economic benefits for the electricity retailer and the ISO. In particular, we demonstrated that a multi-service approach significantly increases retailers' economic benefits compared to solely participating in the energy market, due to the combination of low activation rates and high capacity payments of the reserve market. This combination allows the retailer to attain elevated capacity payments for being on-hold, without necessarily having to utilize its flexible load contracts for reserve deployment during real-time operation.

Regarding the economic benefits perceived by the ISO, we concluded that enabling demand-side participation in the reserve market contributes to an overall reserve cost reduction. This is due to the fact that the retailer has the capability to displace the most expensive generator-based bids of the reserve market and therefore drive down the reserve acquisition costs for the ISO. This phenomena is more prominent when increasing the volume of flexible load contracts available to the retailer; However, we demonstrated that the retailer experiences diminishing returns per MW of contracted volume above a specific threshold, and therefore the total volume of contracted loads is upper-bounded due to not being economically efficient.

Finally, we validated the robustness of our proposed two-stage model by testing two separate sensitivities, where we demonstrated that the retailer has the capability to attain significant economic benefits compared to solely participating in the energy market even in high activation rate scenarios such as $\rho = 40\%$, but at the same time may face diminishing returns per MW of contracted flexible loads upon a specific threshold.

5.2 Further Work

Loads modeled in the proposed two-stage methodology are only capable of providing DR through load curtailment (i.e. reduction of energy consumption). Thus, it would be very interesting to expand the number of DR services offered by flexible loads beyond curtailment so that the potential of DR could be fully studied. Such DR services may include the ability to perform load shifting and/or increase consumption during periods of low demand, which will be particularly relevant when managing RES variability in future low-carbon systems.

Additionally, the number of demand-side resources can be expanded beyond flexible loads, with the inclusion of demand-side technologies such as distributed generation (DG), distributed storage (DS) and electric vehicles (EV). This will allow for the analysis of the complete spectrum of demand-side solutions and therefore study the economic potential of each resource, and how the retailer can integrate different demand-side resources into their portfolio.

Regarding market participation, this work only considered that the retailer can utilize flexible loads to offer DR-based services in energy and reserve markets. Therefore, one topic that could be further expanded corresponds to increasing the number of markets where the retailer can offer DR-based services. It would interesting to analyze how the integration of additional markets, such as load interruption programs or capacity markets can have an influence on the multi-service business case for the retailer and what are the most profitable services to participate in. As a consequence, by expanding the number of services provided by the retailer in electricity markets and analyzing the most profitable markets to offer its flexibility, market-based conclusions could be drawn from the ISO's perspective to determine which markets have the highest potential for large-scale demand-side participation and what are the impacts on market conditions upon enabling these type of resources in a wide array of different electricity markets.

In addition to market participation, market and price uncertainties can also be integrated in the two-stage framework to provide a robust perspective on future operating scenarios. Currently, the proposed model assumes full knowledge of generator-based bids in the reserve market. This allows the retailer to have complete certainty as to when is the best time to participate in the reserve market, and what is the necessary price-quantity bid that maximizes its own profit compared to other market participants. Therefore, it would be an interesting topic to assume that the retailer does not know a priori the value of generator-based bids, and analyze how this market uncertainty affects the decision-making process of the retailer and its expected benefits in the market. Additionally, other types of market-based uncertainties such as day-ahead prices are also proposed as further research.

Regarding the case studies presented in this work, the validation of the first stage was proposed as a method to determine the forecasting capabilities of our IO-based approach. However, one must acknowledge that even though the model has better forecasting capabilities than other commonly used tools (such as ARX, SVM and regular IO), the tools chosen as comparison are fairly standard and do not integrate the latest advances in data-driven forecasting methods. Since the model has strong performance when compared to standard forecasting tools, it is proposed as further research to expand the comparison into more advanced realms of forecasting. For example, state-of-the-art Machine Learning (ML) algorithms such as Reinforcement Learning (RL) [69] and Deep Learnng (DL) (Convolutional Neural Networks, Recurrent Neural Networks and Autoencoders/Transformer-based models) [70, 71, 72] can be useful benchmarks when analyzing the forecasting capabilities of the proposed IO-method.

In particular, RL can become an attractive alternative due to the large-scale availability of highly granular data. This approach seeks to determine how an agent makes sequential decisions in an environment to maximize the cumulative reward [73]. Mathematically, this decision-making problem is modeled as a Markov Decision process (MDP), which is characterized by a state space, $\mathcal{S}$, action space $\mathcal{A}$, the probability of each next state given a state-action pair (s, a), $\mathbb{P}(\cdot|s, a)$ and a reward function $r(s, a)$. The goal is to find an optimal policy $\pi^*(a|s)$ (what action to make given a certain state) that maximizes expected reward [73]. In the context of this work, we can characterize the decision-making of each flexible load as the following MDP:

- **State:** $s(t) := \{z_{\mathrm{cdt}}\}$, which corresponds to feature vector defined in Section 3.1.3 that contains external variables such as contract price, weather conditions, etc.
- **Action:** $a(t) := \{x_{\mathrm{cdt}}\}$, which corresponds to the energy consumption of each flexible load.
- **Reward:** Cost function of flexible loads' based on its predefined contract with the retailer.

Where several algorithms can be used to determine the optimal policy $\pi^*(a|s)$ (such as Q-learning or Policy Gradient-based algorithms) and forecast future operating scenarios. Therefore, by integrating machine learning algorithms as benchmarks, our IO-based model can be compared against state-of-the-art forecasting methodologies instead of simple and commonly used methods.

Finally, two modeling improvements are proposed to provide more flexibility in retailers' decision-making process (second stage). Currently, equations (3.17)-(3.21) impose a mutually exclusive behavior on retailers' ability to participate in energy market. The DR reduction capability can be used at any given time either in the energy market or in the reserve market, but not in a combination of the two:

$$\Delta_{\mathrm{dt}}^{bid} \leq \sum_{c \in \mathcal{C}} \overline{\Delta}_c \, \phi_{\mathrm{cd}t}^{bid}; \forall \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{5.1}$$

$$\Delta_{\mathrm{cd}t}^{DR} \leq \overline{\Delta}_c \, \phi_{\mathrm{cd}t}^{DR}; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{5.2}$$

$$\phi_{\mathrm{cd}t}^{DR} + \phi_{\mathrm{cd}t}^{\mathrm{bid}} \leq 1; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{5.3}$$

$$\sum_{t \in \mathcal{T}} \phi_{\mathrm{cd}t}^{DR} + \sum_{t \in \mathcal{T}} \phi_{\mathrm{cd}t}^{\mathrm{bid}} \leq 1; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D} \tag{5.4}$$

$$\sum_{\mathrm{d} \in \mathcal{D}} \sum_{t \in \mathcal{T}} \phi_{\mathrm{cd}t}^{DR} + \rho \sum_{\mathrm{d} \in \mathcal{D}} \sum_{t \in \mathcal{T}} \phi_{\mathrm{cd}t}^{\mathrm{bid}} \leq \overline{\phi}_c; \forall c \in \mathcal{C}, \tag{5.5}$$

Therefore, the first proposed modification eliminates this mutually exclusive behavior and allows the retailer to utilize the same flexible load contract throughout different markets:

$$\Delta_{\mathrm{dt}}^{bid} \leq \sum_{c \in \mathcal{C}} (\overline{\Delta}_c - \Delta_{\mathrm{cdt}}^{DR}) \, \phi_{\mathrm{cdt}}^{\mathrm{bid}}; \forall \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{5.6}$$

$$\Delta_{\mathrm{cd}t}^{DR} \leq \overline{\Delta}_c \, \phi_{\mathrm{cdt}}^{DR}; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, t \in \mathcal{T} \tag{5.7}$$

$$\sum_{t \in \mathcal{T}} \phi_{\mathrm{cd}t}^{DR} + \sum_{t \in \mathcal{T}} \phi_{\mathrm{cd}t}^{\mathrm{bid}} \leq 1; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D} \tag{5.8}$$

$$\sum_{\mathrm{d} \in \mathcal{D}} \sum_{t \in \mathcal{T}} \phi_{\mathrm{cd}t}^{DR} + \rho \sum_{\mathrm{d} \in \mathcal{D}} \sum_{t \in \mathcal{T}} \phi_{\mathrm{cd}t}^{\mathrm{bid}} \leq \overline{\phi}_c; \forall c \in \mathcal{C}, \tag{5.9}$$

By modifying constraint (5.1) and eliminating constraint (5.4), the retailer can now freely decide on which market to participate, while also having the capacity to split its contracted flexibility between different markets.

However, since there is a multiplication between a binary variable with a continuous variable ($\Delta_{\mathrm{cd}t}^{DR} \, \phi_{\mathrm{cd}t}^{\mathrm{bid}}$), a new big-M constraint will need to be integrated to reduce the computational burden of the model:

$$\Delta_{\mathrm{cd}t}^{DR} - (1 - \phi_{\mathrm{cd}t}^{\mathrm{bid}}) \, M \leq z_{\mathrm{cdt}} \leq \phi_{\mathrm{cdt}}^{\mathrm{bid}} \, M, \tag{5.10}$$

The second modeling modification aims to increase the number of flexible contract that can be activated by the retailer in a day. Currently the model can only activate its flexible load contracts once per day. Hence, by modifying constraint (5.4), we proceed to relax the daily utilization of flexible load contracts by incorporating a predefined parameter $\nu_c \in \{1...24\}$ that indicates the number of times a contract can be activated during a day:

$$\sum_{t\in\mathcal{T}} \phi^{DR}_{\mathrm{cd}t} + \sum_{t\in\mathcal{T}} \phi^{\mathrm{bid}}_{\mathrm{cd}t} \leq \nu_c; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, \tag{5.11}$$

In addition, the activation parameter ρ can be integrated into constraint (5.5), so that bids in the reserve market are also being accounted for in the daily operation of the retailer:

$$\sum_{t\in\mathcal{T}} \phi^{DR}_{\mathrm{cd}t} + \rho \sum_{t\in\mathcal{T}} \phi^{\mathrm{bid}}_{\mathrm{cd}t} \leq \nu_c; \forall c \in \mathcal{C}, \mathrm{d} \in \mathcal{D}, \tag{5.12}$$

Chapter 6

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